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Abstract

This paper presents a classical-Keynesian one sector model of labor-constrained growth that explains secular stagnation as the result of structural change. Structural change is defined as an exogenous increase in the employment share of stagnant activities, which exhibit no or low labor productivity growth. We discuss two models: (i) a classical distributive cycle in employment rate and labor share, and (ii) a Keynes-Kalecki distributive cycle that adds the income-capital ratio as state variable. Both versions consider labor productivity growth as endogenous to the labor share, reminiscent of induced technical change. Further, growth rates of labor productivity and real wages are assumed to respond negatively to structural change as proxied by the employment share of stagnant activities. Drawing on seminal theories of structural change, we label the positive (negative) difference between these effects dominant Lewis (Baumol) dynamics. In steady state, and across all model variants, the adverse effect of structural change on labor productivity leads to stagnation. However, only the Keynes-Kalecki version with dominant Lewis dynamics and a weak profit squeeze also exhibits a falling labor share.

Keywords: Goodwin cycle; stagnation; structural change; reserve army.
JEL Classification: E12, E25, E32, O41.

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1 Introduction

This paper presents a classical-Keynesian model of labor-constrained growth that explains secular stagnation as the result of structural change. The paper focuses on the formal development of theory, but in this introduction we seek to contextualize our approach and results vis-à-vis the key issues and relevant literature. We proceed in three steps. First, we briefly outline stylized facts. Second, we connect these to theoretical and empirical literature on labor suppression and secular stagnation. Third, we motivate the contribution of this paper: to provide a macroeconomic model consistent with the stylized facts that renders structural change an additional cause of secular stagnation.

We begin with stylized facts, with a focus on the post-war US economy. First, there are cyclical stylized facts: macroeconomic activity variables lead the labor share, or, equivalently, cycles in employment rate-labor share and income-capital ratio-labor share space are counter-clockwise. These patterns are consistent with profit led activity and profit squeeze distribution and hence consistent with neo-Goodwinian perspectives (Goodwin, 1967; Flaschel, 2015; Barrales et al., 2021b).

Second, the US macroeconomy has (and other advanced economies have) undergone crucial long run changes since the beginning of the neoliberal era around 1980: the labor share, the income-capital ratio, the growth rate of labor productivity and hence the natural rate of growth as well as the observed average rate of growth of real GDP have declined. Employment statistics show a mixed picture: while the US employment rate has not surpassed its peak during the new economy boom, the unemployment rate (pre-Covid) reached record lows. Further, the employment share of sectors with zero or very low labor productivity growth—what we will call *stagnant activities*[1]—has increased significantly.[2]

Classical and (post-)Keynesian approaches are well suited to tackle these sets of

[1] In his seminal paper, Baumol (1967, p. 415) starts with the "basic premise that economic activities can, not entirely arbitrarily, be grouped into two types: technologically progressive activities in which innovations, capital accumulation, and economies of large scale all make for a cumulative rise in output per man hour and activities which, by their very nature, permit only sporadic increases in productivity." Throughout, we distinguish between progressive and stagnant sectors or activities in this sense.

[2] Many collections of these stylized facts have been put together, and they are largely uncontroversial. For examples, see Petach and Tavani (2020, Fig. 1, p. 237) or Kiefer et al. (2020, Fig. 1, p. 194). A key issue regarding potential or natural growth is that the decline precedes the *Great Recession*. The prime age employment rate had its high point at 81.9% in April of 2000; neither of the two business cycle peaks since surpassed 81%. The civilian unemployment rate stood at 3.5% in February of 2000, at lows not seen since the late 1960s. See Federal Reserve Economic Database (FRED) series LREM25TTUSM156S and UNRATE, respectively. Mendieta-Muñoz et al. (2020, Fig. 3) show sectoral data: employment shares of low productivity activities such as professional business services, health, education, and arts, entertainment and recreation have roughly tripled in recent decades, whereas those of high productivity activities such as manufacturing, wholesale trade and information technology have declined.

stylized facts on cycles and trends in combination.[3] These literatures emphasize systematic interaction between growth and the functional distribution of income. Moreover, there exists a strong tendency to see the process of growth as resulting from the cycle itself. Nonlinearities or at least strong propagation mechanisms give the business cycle a central role in the explanation of growth. This approach has led to a promising synthesis of short and long run narratives.[4] In particular, recent research suggests that short run demand (and employment) is profit led, but that the long run features wage led productivity growth. Put differently, the cycle is of Goodwin type, but the natural rate of growth in steady state is positively related to the labor share. This link opens the door to explain observed tendencies for stagnation in conjunction with—or, importantly, caused by—a worsening of the functional distribution of income.

The possibility arises due to an assumed positive effect of the labor share on labor productivity growth. Kennedy (1964) provides a microeconomic justification for this mechanism of induced or directed technical change. Shah and Desai (1981) incorporate the framework in the classical distributive cycle; Tavani and Zamparelli (2017) survey recent literature in this vein. Taylor (2004) and Storm and Naastepad (2012) motivate labor productivity as a macroeconomic aggregate, and include it as increasing in the labor share in models with Keynesian and Kaleckian features. All of this literature ultimately draws on Marx (1867), in particular chapters 15 and 25.

In such models, labor suppression can lead to secular stagnation in steady state even if the short run remains profit led. Petach and Tavani (2020) presents such an effort in a model that also includes dynamics of wealth distribution. The authors build on a micro-founded Kennedy-style framework, which requires to assume that the exogenous shock affects the intercept of the innovation possibility frontier. Michl and Tavani (2021)discuss classical models that also feature a positive causal connection between labor share and growth in steady state. Rada et al. (2021) derive similar results in models of the distributive cycle with Keynesian features: an adverse shock to the labor share leads in the short run to an increase in accumulation due to its profit led character. In steady state, however, realized and warranted rate of growth have to converge to the natural rate of growth, which has decreased due to lessened pressure to innovate and save on labor costs.

Recent empirical papers that investigate the long run connection between growth

[3] We will not review the burgeoning neoclassical and new Keynesian literature on potential causes of secular stagnation. These theories build on marginal productivity theory, which links income shares to the elasticity of substitution in CES-technology. To match stylized facts, it must be assumed that this elasticity is larger than unity—which is not supported by evidence. See Teulings and Baldwin (2014) for relevant ideas and references.

[4] The classical notion of gravitation and a (post-)Keynesian emphasis on endogenous cycles are one theoretical side of this coin, the other is the clean empirical distinction between short and long run issues. See the introduction of Barrales and von Arnim (2021) for a discussion of and references on the former, and Blecker (2016) for an important motivation of the latter.

and distribution conditional on the short run cycle include Kiefer and Rada (2015) and Kiefer et al. (2020). The former estimate an OECD panel for the neoliberal era that exhibits Goodwin dynamics, and also suggest the possibility of a positive link between output gap and labor share in the long run. The latter estimate the growth rate of real potential output in the post-war US conditional on the trend decline in the labor share, and short run interactions between growth and labor share. Results are consistent with the perspectives outlined here. Barrales et al. (2021b) present a survey of the empirical evidence on distributive cycles, including suggestive but as of yet inconclusive evidence on the long run interaction between measures of activity and distribution in the frequency domain (see Section 4.2 of that paper).

The contribution of the present paper is to connect these themes to the topic of structural change. The connection arises very intuitively: as economies mature, the share of employment in sectors with no or low productivity growth—i.e., stagnant activities—rises. This implies a compositional adverse effect on labor productivity. We label this negative response of the growth rate of labor productivity to the employment share of stagnant activities the *productivity effect*. Importantly, this productivity effect (*ceteris paribus*) places upward pressure on the labor share. Further, however, the level of real wages in stagnant activities is significantly lower than in progressive activities. A shift of employment from progressive towards stagnant activities can therefore be expected to undermine bargaining power of labor and hence the growth rate of real wages. We label this the *real wage effect*. Low productivity activities act as a sink for the reserve army, or surplus labor, which keeps the aspirations of labor employed in progressive activities in check. *Ceteris paribus*, the real wage effect places downward pressure on the labor share.

In the models considered here, the difference between productivity and real wage effect turns out to be crucial. Drawing on Lewis (1954) and Baumol (1967), we label this difference as *dominant Lewis dynamics* if positive, and *dominant Baumol dynamics* if negative. In summary, structural change of the type considered here implies a decline in aggregate productivity growth, *and* a decline in real wage growth. The difference between productivity effect and real wage effect is positive (negative) if labor market institutions are as in Lewis's (Baumol's) theory.

Current empirical research supports these ideas. In particular, Taylor and Ömer (2020) use shift-share methodology to decompose the US labor share across sectors. Based on results, the authors hypothesize a "reverse Lewis" shift, since Lewis's focus on development meant to consider expansion of progressive sector employment. Mendieta-Muñoz et al. (2020) utilize a Divisia index decomposition for the US labor share across sectors. Results provide support for an adverse compositional productivity effect: sectors with low productivity and pay have relatively high labor shares, so that the shift towards stagnant activities puts upward pressure on the labor share. However, as this shift occurs, the contribution from real compensation in progressive activities towards the aggregate labor share fails to keep up with the contribution from labor productivity. This is designated as "downward decoupling." Downward pressure on the labor share in progressive sectors exceeds the upward pressure in stagnant sectors, suggesting that Lewis dynamics prevail. Storm (2018) theorizes along these lines that a re-dualization of the US economy is a causal factor in stagnationary tendencies.

Barrales et al. (2021a) offers a theoretical discussion of the stylized facts regarding neo-Goodwinian growth cycles, secular stagnation and the sectoral narratives—reverse-Lewis, or re-dualization, or progressive sector downward decoupling. Our objective here is to fill that narrative with formal detail. To keep things simple, we discuss one sector models and introduce sectoral effects via an exogenous parameter. The research question is under what theoretical closures and parameter constraints a neo-Goodwinian model conforms to all relevant stylized facts *and* generates secular stagnation as a result of structural change. None of this is to argue that other candidate causes—labor suppression, globalization, financialization, and macroeconomic demand management with a contractionary bias—do not play a (possibly more important) role. Our argument merely is that structural change can be seen as an additional causal factor, specifically through the here proposed linkages with labor market institutions.

The remainder of the paper is organized as follows. The next section motivates productivity effect and real wage effect in light of Harrod's three growth rates. Section 3 discusses a two-dimensonial classical distributive cycle in employment rate and labor share, and Section 4 a three-dimensional Keynes-Kalecki distributive cycle that adds the income-capital ratio as state variable. Both versions consider labor productivity growth as endogenous to the labor share, reminiscent of induced technical change. To foreshadow results: in steady state, and across all model variants, the productivity effect leads to stagnation. However, only the Keynes-Kalecki version with dominant Lewis dynamics and a weak profit squeeze also exhibits a falling labor share.

2 Baumol vs. Lewis: Labor markets, labor productivity, and natural growth

This section provides an overview of the topic of structural change, and how we seek to introduce it further below in the formal models. We start the section with a review of Harrod's three growth rates, to fix notation and connect the stylized facts already mentioned in the introduction to this standard framework.[5] Second, we motivate productivity and real wage effects, and Baumol and Lewis dynamics in detail, and through these channels link structural change to the natural rate of growth. We

[5]Though Harrod's growth rates provides a foundation, we do not address Harrodian instability issues, which could imply the emergence of a limit cycle. See also footnote 15.

close with a summary of our key results from both the classical distributive cycle and the Keynes-Kalecki version.

In steady state, Harrod's three growth rates are equal:

$$g = s_\pi(1-\psi)u = a + n, \tag{2.1}$$

where $g = I/K = \hat{Y}$ is the realized rate of growth (with K non-depreciating capital stock, $I = \dot{K}$ and Y the level of real GDP), the second term is the warranted rate of growth g^w, and the third is the natural rate of growth g^*. s_π is the exogenously given savings propensity of capitalists.[6] $\pi = (1-\psi) = 1 - wL/PY$ is the profit share, and ψ the labor share. $u = \sigma U = (Y^*/Y)(Y^*/K) = Y/K$ is the income-capital ratio and the product of full capacity output to capital ratio σ and utilization rate U. We define the growth rates of average labor productivity $\hat{A} = \hat{Y} - \hat{L} = a$ and labor force $\hat{N} = n$, so that $g^* = a + n$.

Now recall the stylized facts of secular stagnation during recent neoliberal decades: income-capital ratio, labor share and steady state rate of growth have all decreased; $u^* \downarrow, \psi^* \downarrow, g^* \downarrow$. Further, the employment rate has increased or at least remained high; $e^* \uparrow$. As long as a and n are fully exogenous "manna from heaven," neoclassical supply-side visions of stagnation à la Gordon (2016) are the only feasible way forward.[7] The natural rate declines due to exhaustion of useful innovations and inexorable demographic trends. Crucially, warranted and realized rates of growth adjust to g^*, and g^w must do so Piketty (2013)-style: a larger than unity elasticity of substitution makes a decline in ψ and corresponding rise in capital-income ratio possible. However, if the growth rate of labor productivity is an increasing function of the labor share, labor suppression leads to a decline in the labor share and an endogenous response of the natural growth rate in the same direction. This is the crux of the argument for a classical-Keynesian theory of inequality-driven secular stagnation, laid out in various versions in the literature (Petach and Tavani, 2020; Michl and Tavani, 2021; Barrales et al., 2021a; Rada et al., 2021).

Here we extend the investigation to structural change, and now introduce the productivity effect and real wage effect. To do so, let us first motivate the seminal theories of structural change put forth in Baumol (1967) and Lewis (1954). Both consider an economy with two sectors, one of which is progressive (i.e. featuring high labor productivity growth) and one of which is stagnant (i.e. featuring no or low labor productivity growth). Baumol wrote about the vicious implications of structural change from progressive manufacturing toward stagnant service activities, while Lewis worked on the virtuous possibilities of structural change from stagnant

[6]Throughout, and as is standard, for any variable x, $\dot{x} = dx/dt$ is the time rate of change and $\hat{x} = \dot{x}/x$ is the proportional growth rate.

[7]See also Michl and Tavani (2021, Section 7) for a discussion of these supply side factors in contrast to structuralist narratives, i.e. the pertinent "political and institutional contradictions."

agricultural to progressive manufacturing activities. For the purposes of our discussion, we do not want to emphasize these differences, which appear predicated merely by their specific interests and time of writing. Instead, our focus lies on the very different treatment of labor markets.

In Baumol's theory, real wages in stagnant activities are required to grow at the rate of aggregate productivity growth, to continually attract labor in a competitive labor market. In sharp contrast, in Lewis's theory, jobs in the progressive sector are limited, and while its real wages feature a significant premium, they remain depressed by the existence of a pool of underemployed—a "reserve army of labor"—in the stagnant sector (see also Mendieta-Muñoz et al., 2020, Section 6). This juxtaposition is illustrative. In Baumol, productivity gains are necessarily and broadly shared. Otherwise, producers could not hire labor to satisfy consumption demand. In Lewis, labor markets are dual. Stagnant sector jobs are not as desirable, productive and well paid as progressive sector jobs, but the latter are scarce and the fallback option is unemployment and poverty. In short, labor market institutions define the difference between Lewis's and Baumol's theory.

To further frame the issue, consider working the ticket booth at the local cinema vs. manual labor in a manufacturing plant, and consider that the former type of employment opportunity proliferates. Clearly, it will undermine labor's bargaining power in remaining progressive activities. We define structural change as such a proliferation of relatively unproductive and less well paid employment opportunities—in other words, an increase in the employment share of stagnant activities, $\lambda \uparrow$. Specifically,

$$\omega_\lambda < 0 \Leftrightarrow \text{real wage effect.} \tag{2.2}$$

Ceteris paribus, the assumed employment shift adversely affects the position (or slope) of the real wage Phillips curve. Second, the productivity effect is compositional: the shift towards activities with low(er) innovation potential reduces the average rate of labor productivity growth:

$$a_\lambda < 0 \Leftrightarrow \text{productivity effect, and} \tag{2.3}$$

In summary, both effects are motivated on the basis of sectoral interactions—but are introduced here for the *aggregate* growth rates of labor productivity a and real wages ω.

The following inequalities specify the difference between labor productivity effect 2.3 and real wage effect 2.2 as

$$a_\lambda - \omega_\lambda > 0 \Leftrightarrow \text{ dominant Lewis dynamics, and} \tag{2.4}$$

$$a_\lambda - \omega_\lambda < 0 \Leftrightarrow \text{ dominant Baumol dynamics.} \tag{2.5}$$

We are now ready to state the productivity rule:

$$a = a(\psi; \lambda), a_\psi > 0, a_\lambda < 0, \tag{2.6}$$

	2D model, Section 3		**3D model**, Section 4	
	Baumol	Lewis	Baumol	Lewis
Income-capital ratio u^*	n.a.	n.a.	$+/-$	$-$
Employment rate e^*	$+/-$	$+$	$-$	$+$
Labor share ψ^*	$+$	$+$	$+$	$+/-$
Natural rate of growth g^*	$-$	$-$	$-$	$-$

Table 1: Comparative dynamics. The table summarizes results of comparative dynamic exercises. The exogenous change is an increase in the share of employment in stagnant activities (λ). The first two columns report results for the classical model of Section 3, the last two columns those for the Keynes-Kalecki version with an endogenous income-capital ratio of Section 4. *Baumol* indicates that Baumol dynamics dominate, i.e. $a_\lambda - \omega_\lambda < 0$, and *Lewis* that $a_\lambda - \omega_\lambda > 0$; see Section 2 for discussion.

where $a_\psi > 0$ captures the "induced technical change" effect, and $a_\lambda < 0$ the productivity effect. The natural growth rate follows as $g^* = a(\psi; \lambda) + n$, and its derivative with respect to the stagnant sector employment share is

$$\frac{\partial g^*}{\partial \lambda} = a_\psi \frac{\partial \psi^*}{\partial \lambda} + a_\lambda. \tag{2.7}$$

Given the signs of these partials, g^* always declines in response to structural change when $\partial\psi^*/\partial\lambda < 0$. However, in a macroeconomic model the general equilibrium effects depend on more than one equation, and we conclude this section now with a summary of effects in steady state.[8] Key results are summarized in Table 1.

Importantly, both model versions show a decline of the natural (and hence warranted and realized) rate(s) of growth in steady state in response to an increase in λ. In other words, the direct (compositional) productivity effect via a_λ takes center stage. The indirect induced technical change effect $a_\lambda \partial\psi^*/\partial\lambda$, in contrast, carries less weight.[9] This is not obvious *prima facie*, especially since no further assumptions on the magnitude of the relevant coefficients are made. Instead, it follows merely from the structure of the model(s) and the assumed signs. It appears nevertheless intuitive: it would seem unlikely to obtain a higher natural rate of growth due to cost-saving pressures despite ongoing tertiarization, rise in labor-intensive activities, and increase of unproductive employment.

The second key result is that only the Keynes-Kalecki distributive cycle model of Section 4 can generate the positive link between labor share and natural rate of growth in steady state. The first three columns of Table 1 all report a steady state increase in the labor share. The fourth column indicates that the three-dimensional model with a Lewisian labor market can exhibit either a rising or falling labor share.

[8]Subsequent sections and appendices provide all of the formal detail.

[9]The labor share effect is important, however, in buffering the decline of the steady state growth rate in the case of Baumol, relative to the case of Lewis.

A key condition for $\partial\psi^*/\partial\lambda < 0$ is a relatively weak or weakening profit squeeze (ω_e in equation 3.4 below), which appears to be a key feature of recent decades.[10]

Moreover, the simple two-dimensional classical model put forward here does not speak to the income-capital ratio, since σ is constant. In Shah and Desai (1981), the income-capital ratio is constant in steady state, but converges to that equilibrium in response to technological changes. In particular, σ rises in the natural rate of growth and decreases in the savings rate $s = s_\pi \pi$. Since $a_\psi > 0$, the steady state income-capital ratio increases in the labor share, which in turn is pinned down by the curvature of the innovation possibility frontier. Petach and Tavani (2020) build on this link to explain the "Piketty fact" of a rising capital-income ratio and a falling labor share in response to an adverse shock to labor market institutions. Though causal mechanisms focus tightly on technology, the theory does not require Humbug production functions. As the top row of Table 1 indicates, our Keynes-Kalecki version matches the falling income-capital ratio also, but relies on the mechanism of the paradox of thrift (see also footnote 12).

In summary, the models of the distributive cycle discussed in this paper produce secular stagnation *and* a falling labor share in response to a rise in the employment share of stagnant activities if (i) the income-capital ratio responds to a Keynesian expenditure function, (ii) labor markets are Lewisian, and (iii) the profit squeeze is relatively weak. The neo-Goodwinian context implies that the model remains labor constrained, since steady state growth is limited by the growth of the effective labor force. However, (ii) and (iii) in combination imply that labor simultaneously experiences diminishing opportunities, and a significant decrease in its power to effect real wage increases.

3 A Goodwin model with endogenous labor productivity growth and structural change

In this section, we discuss a Goodwin model with classical features: the model's two state variables are employment rate and labor share, the income-capital ratio is constant, and all available savings are channeled into capital accumulation. We deviate from the original in Goodwin (1967) in that the savings rate of capitalists is less than unity, and the growth rate of labor productivity is not constant, but a positive function of the labor share. As a consequence, the steady state labor share

[10]The complete additional parameter constraints are discussed in Section 4 in detail. Mendieta-Muñoz et al. (2020) find a weakening profit squeeze in an empirical application of a neo-Goodwinian model. Setterfield (2021) discusses the weakening profit squeeze in the context of Goodwinian theory, and questions its enduring relevance. It seems clear that the Goodwin pattern has not disappeared, though it is also unquestionably true that flattening Phillips curves are a fact of neoliberal life. For a new-Keynesian empirical perspective that evokes the possibility of relatively flat and likely non-accelerationist Phillips curves, see Blanchard (2016).

is increasing in the savings propensity. Put differently, a declining propensity to invest (as proxied by the savings rate in a model with this version of Say's Law) can depress the steady state growth rate *and* the labor share.

The latter links natural rate of growth and the distribution of income. The productivity growth function assumed here is reminiscent of Kennedy (1964), but foregoes the microeconomic apparatus and hence does not require the introduction of an innovation possibility frontier and third state variable. But as in Shah and Desai (1981), the model converges in a stable focus, since capitalists have an "additional weapon" and the pair of complex eigenvalues are not any longer purely imaginary. As in other classical models, the distribution of income remains disconnected from bargaining parameters. Instead, the labor share is fully determined by technology.[11] It follows that in steady state, the productivity effect always outweighs the real wage effect, *even if Lewis dynamics prevail*, i.e. $a_\lambda - \omega_\lambda > 0$. Specifically, ω_λ has no effect in steady state, but a_λ decreases labor productivity and hence increases the labor share.

We include this discussion here since it allows us to introduce productivity and real wage effects in the context of a simple macro model. In summary, the two-dimensional classical distributive cycle presented here always generates stagnation in response to structural change, *but the labor share increases*. This means that the classical model is inconsistent with structural change in isolation as a causal factor for observed stylized facts. The three-dimensional version in Section 4 resolves this tension.

3.1 The model

The classical distributive cycle features two laws of motions:

$$\dot{e} = e(g - (a + n)) \tag{3.1}$$

$$\dot{\psi} = \psi(\omega - a), \tag{3.2}$$

where employment rate $e = L/N$ and labor share $\psi = wL/PY$ are the state variables; $\hat{\Omega} = \omega$ is the growth rate of real wages. All other items were defined previously.

These laws of motions are brought to life with warranted rate of growth with $\sigma = Y^*/K$ the constant full capacity output to capital ratio, a behavioral function to describe endogenous labor productivity (reprinted here from the previous section), and a behavioral function to describe real wage growth:

$$g = s_\pi(1 - \psi)\sigma \tag{3.3}$$

[11] Petach and Tavani (2020) solve this problem by assuming that shocks to labor market institutions affect the steady state labor share *and* the intercept of a linearized innovation possibility frontier.

$$a = a(\psi; \lambda), a_\psi > 0, a_\lambda < 0 \tag{2.6}$$

$$\omega = \omega(e; \lambda), \omega_e > 0, \omega_\lambda < 0, \tag{3.4}$$

where λ is the employment share in stagnant activities, ω_e the profit squeeze parameter, and other relevant partials were motivated previously in Section 2.

The nonlinear two-dimensional system of differential equations is given by substitution of the last three equations into the two laws of motion. The Jacobian matrix at the non-trivial steady state follows as:

$$J^* = \begin{bmatrix} 0 & -e(s_\pi \sigma + a_\psi) \\ \psi\omega_e & -\psi a_\psi \end{bmatrix}, \tag{3.5}$$

where we refrain from starring variables for brevity.

The sign pattern of this matrix is unambiguous. $Tr(J^*) < 0, |J^*| > 0$ guarantee asymptotic stability: the model converges in a stable focus to the non-trivial steady state. These dynamics differ from Goodwin's original conservative oscillation. Here, the productivity growth rule mediates the perpetual predator-prey cycle. Nevertheless, the dampened cycle is of Goodwin type, which implies that the employment rate leads the labor share. Equivalently, the direction of the stable focus is counter-clockwise in e, ψ-plane. In a phase diagram in e, ψ-plane, the $\dot{\psi} = 0$ isocline is upward sloping (rather than vertical as in original Goodwin), whereas the $\dot{e} = 0$ isocline continues to be horizontal. The latter also implies that the steady state distribution of income is invariant to bargaining parameters, and instead fully determined by technological constraints.

3.2 Structural change

This subsection presents comparative dynamic exercises for the two-dimensional model presented above. We list below the effects of an increase in the stagnant sector employment share λ on the state variables e and ψ and also the natural growth rate g^*. We apply Cramer's rule; see Appendix A.1 for derivations.

$$\begin{aligned} \frac{\partial e^*}{\partial \lambda} &> 0 \Leftrightarrow a_\lambda - \omega_\lambda > 0 \\ &\gtrless 0 \Leftrightarrow a_\lambda - \omega_\lambda < 0 \end{aligned} \tag{3.6}$$

$$\frac{\partial \psi^*}{\partial \lambda} = -\frac{a_\lambda}{s_\pi \sigma + a_\psi} > 0 \tag{3.7}$$

$$\frac{\partial g^*}{\partial \lambda} = a_\lambda \left(1 - \frac{a_\psi}{s_\pi \sigma + a_\psi}\right) < 0 \tag{3.8}$$

Table 1 also summarizes these results.

First, the employment rate always increases in λ if Lewis dynamics are dominant. The employment rate also increases with dominant Baumol dynamics if ω_λ and

a_ψ are relatively strong, and vice versa: if the growth rates of real wages and labor productivity are *not* strongly elastic, the dominant Baumol dynamics imply a decline in the employment rate.

Second, in this classical model of the distributive cycle, the labor share always rises with an increase of λ. The reason is the invariance of the functional distribution of income to real wage Phillips curve parameters: neither ω_e nor ω_λ matter for the steady state labor share—but a_λ does, and the decline in productivity puts upward pressure on ψ.

Third, the natural rate of growth in steady state responds negatively to an increase in stagnant sector employment share. This is always the case, even though equation 2.7 would suggest that $\partial\psi^*/\partial\lambda > 0$ matters. However, in steady state, only a_λ and a_ψ have an impact, and the partial can be signed unambiguously. Further, the decline of g^* is more pronounced if labor markets are Lewisian, since the fall in the labor share presents additional drag.

4 A Keynes-Kalecki distributive cycle with endogenous labor productivity growth and structural change

The model of this section removes the assumption of Say's Law. The income-capital ratio becomes third state variable, and responds to an independent expenditure function. In consequence, model characteristics change in important ways. First, the positive association between growth and savings reverses. Where capitalists' savings propensity proxies the willingness to invest in productive assets in the classical version, it now represents a leakage. Thus, the introduction of the independent expenditure function also implies the paradox of thrift. In this model, the paradox of thrift applies to the long run, and the income-capital ratio is endogenous to demand in the long run.[12]

Second, the independent expenditure function also removes the invariance of the distribution of income vis-à-vis bargaining parameters. Technically, this is due to the fact that the trace of the Jacobian matrix evaluated at the steady is non-zero everywhere; it is this vanishing of the trace in classical versions of the distributive cycle that de-links the labor share in steady state from labor market institutions as proxied by real wage Phillips curve parameters. More intuitively, the Keynesian

[12] The utilization controversy considers this assumption in detail. Critics of neo-Kaleckian approaches view it as inadmissible to consider deviations from a desired rate of utilization in the long run. Various solutions have been proposed; see Nikiforos (2020) for a useful discussion and further references. The issue takes on new importance in the context of secular stagnation: taking the savings propensity as given, the equality in steady state of Harrod's growth rates requires that the income-capital ratio falls by more than the profit share rises. Otherwise, the warranted rate cannot match the natural rate's decline. In that sense, we merely choose one mechanism to make that possible. See also Rada et al. (2021) for related discussion.

expenditure function introduces slack, which means that the economy is *not* at the efficient frontier and hence *not* primarily constrained by technology.

In combination, these extensions enable the model to describe the stylized facts of the neoliberal era as the result of labor suppression. An adverse shock to labor's bargaining power, proxied by a downward shift of the real wage Phillips curve (or a decline in its slope), leads to a decline in the labor share, the income-capital ratio, the growth rate of labor productivity and hence the natural rate of growth as well as the observed average rate of growth of real GDP. As mentioned in the introduction, the real world employment picture is ambiguous, but the model predicts an *increase* in the employment rate. See Barrales et al. (2021a) for a broader discussion, and Rada et al. (2021) for formal detail.

The purpose of the discussion here is to investigate how structural change fits into the model and these narratives of secular stagnation. Just below we discuss the model and its dynamic and stability properties. The following subsection demonstrates key results: An increase in the stagnant sector employment share ($\lambda \uparrow$) in this three-dimensional Keynes-Kalecki distributive cycle causes secular stagnation ($u^* \downarrow, e^* \uparrow, \psi^* \downarrow, g^* \downarrow$) if Lewis dynamics are dominant ($a_\lambda - \omega_\lambda > 0$) and the profit squeeze (ω_e) is relatively weak.

4.1 The model

The Keynes-Kalecki distributive cycle features three laws of motion.[13] Specifically, the model adds the income-capital ratio $u = Y/K = \sigma U$ as a state variable. The three-dimensional system of nonlinear differential equations is

$$\dot{u} = u(h - g) \tag{4.1}$$

$$\dot{e} = e(h - (a + n)) \tag{4.2}$$

$$\dot{\psi} = \psi(\omega - a). \tag{4.3}$$

The law of motion for the income-capital ratio defines the time rate of change of u as a function of the difference between real growth rates of output $\hat{Y} = h$ and capital stock $g = I/K$. The law of motion for the labor share is as in Section 3, but the law of motion for the employment rate in equation 4.2 now directly references output growth h. In the classical version, in contrast, $g = h$ at all times.

The model includes equations 2.6, 3.3 and 3.4: the productivity rule, the warranted rate of growth, and the real wage Phillips curve. Additionally, it features an independent expenditure function h that determines the growth rate of output:

$$h = h(u, e, \psi), h_u > 0, h_e < 0, h_\psi < 0 \tag{4.4}$$

[13]The model of this section is similar in spirit to Barbosa-Filho (2004) and von Arnim and Barrales (2015, Section 3.2). In contrast to this and related literature, our model here focuses directly on warranted and natural rates of growth, and is otherwise stripped down to emphasize the key steady state linkages via the productivity growth rule.

The partials can be motivated as follows. First, h_u is positive, as in Skott (1989): a higher level of demand as proxied by a higher income-capital ratio u leads to an increase in the growth rate of output. Barbosa-Filho and Taylor (2006) assume $h_u < 0$, which immediately satisfies dynamic stability of the activity variable. However, in Skott's framework and also here, the inclusion of the employment rate requires the assumption that increases in demand lead to increases in the employment rate. Own-stability of the income-capital ratio is still satisfied, as long as $h_u - g_u < 0$.

Second, $h_e < 0$. Skott (1989, p. 236) motivates this sign as a decrease in the desired rate of expansion due to adjustment and turnover costs at high employment rates. The sign can also be motivated with direct reference to Kalecki (1943): high employment rates undermine the power of capital, and thus depress expansion plans (see also Flaschel et al., 2007). Last but not least, $h_\psi < 0$ is the standard neo-Kaleckian link from functional distribution of income to economic activity, although here driving investment as expenditure first. In summary, h is an independent expenditure function that renders the model Keynesian-Kaleckian.

Substituting equations 2.6, 3.3, 3.4 and 4.4 into the laws of motion 4.1–4.3 gives the following Jacobian matrix, evaluated at the non-trivial steady state:

$$J^* = \begin{bmatrix} u(h_u - s_\pi(1-\psi)) & uh_e & u(h_\psi + s_\pi u) \\ eh_u & eh_e & e(h_\psi - a_\psi) \\ 0 & \psi\omega_e & -\psi a_\psi \end{bmatrix} \tag{4.5}$$

We assume $h_u < s_\pi(1-\psi)$ to obtain stable own-feedback of the income-capital ratio, and $|h_\psi| > s_\pi u$ to obtain profit-led demand.[14] The sign pattern follows as

$$J^* = \begin{bmatrix} - & - & - \\ + & - & - \\ 0 & + & - \end{bmatrix}. \tag{4.6}$$

Appendix A.2 lists the Routh-Hurwitz conditions and discusses stability. The first three inequalities are always satisfied, given the assumptions on signs in the Jacobian 4.6. The appendix also provides a sufficient condition for the fourth inequality (eq. A.9) to hold:

$$-(eh_e - \psi a_\psi) > uh_u. \tag{4.7}$$

Assuming that this inequality is satisfied, the three-dimensional model is asymptotically stable. While condition 4.7 is only sufficient, it is straightforwardly interpreted:

[14]We will not relitigate the question of demand regimes: no convincing empirical evidence for short run wage led demand exists. See Basu and Gautham (2019); Barrales et al. (2021b). Relevant research questions regarding the Goodwin pattern exist; these concern the weakening of the profit squeeze and the role of pro-cyclical labor productivity over the course of the cycle (Setterfield, 2021), but not the Goodwin pattern itself.

the stabilizing elements along the trace have to outweigh the destabilizing element h_u. In particular, h_u appears in the law of motion of the employment rate, and there can lead to violation of the fourth Routh-Hurwitz inequality.[15]

Further, the model generates relevant cyclical stylized facts (Zipperer and Skott, 2011; von Arnim and Barrales, 2015; Barrales et al., 2021b). The two-dimensional subsystems are consistent with real world cycles in u, e and e, ψ. The u, ψ-cycle emerges only in the three-dimensional system, and is there determined by $\partial \dot{e}/\partial u = eh_u > 0$: the employment rate increases in the income-capital ratio, and then drives the profit squeeze via ω_e. The model thus conforms to the relevant cyclical stylized facts.

4.2 Structural change

Here we list comparative dynamic exercises for the three-dimensional model. We collect below the effects of an increase in the stagnant sector employment share λ on the state variables u, e and ψ and also the natural growth rate g^*. We utilize Cramer's rule; see Appendix A.3 for derivations.

$$\frac{\partial u^*}{\partial \lambda} < 0 \Leftrightarrow a_\lambda - \omega_\lambda > 0$$

$$> 0 \Leftrightarrow a_\lambda - \omega_\lambda < 0 \text{ and } \frac{\omega_\lambda a_\psi}{s_\pi u a_\lambda} + \frac{\omega_e}{h_e}\left(1 + \frac{h_\psi}{s_\pi u}\right) < \frac{(a_\lambda - \omega_\lambda)}{a_\lambda} \tag{4.8}$$

$$\frac{\partial e^*}{\partial \lambda} > 0 \Leftrightarrow a_\lambda - \omega_\lambda > 0$$

$$< 0 \Leftrightarrow a_\lambda - \omega_\lambda < 0 \tag{4.9}$$

$$\frac{\partial \psi^*}{\partial \lambda} < 0 \Leftrightarrow a_\lambda - \omega_\lambda > 0 \text{ and } \frac{a_\lambda - \omega_\lambda}{a_\lambda} < \frac{\omega_e}{h_e} < 0$$

$$> 0 \Leftrightarrow a_\lambda - \omega_\lambda < 0 \tag{4.10}$$

$$\frac{\partial g^*}{\partial \lambda} < 0 \tag{4.11}$$

Recall that $a_\lambda - \omega_\lambda > 0$ implies that Lewis dynamics are dominant. Table 1 also summarizes these results.

The income-capital ratio can increase or decrease if Baumol dynamics dominate. However, if the real wage effect is stronger than the productivity effect, the income-

[15]The fourth Ruth-Hurwitz inequality A.9 ensures that the real parts of a potential pair of complex eigenvalues are negative. In numerical simulations, we confirmed that increases in h_u lead to a Hopf bifurcation as the real parts pass through zero from below. A stable limit cycle emerges (with linear behavioral functions). The assumption of asymptotic stability greatly simplifies comparative dynamic exercises, which are our focus here. This result, however, suggests that the model could plausible generate endogenous fluctuations, which is appealing for a theory of cyclical growth. Details are available upon request. See Barrales and von Arnim (2021) for a related investigation.

capital ratio always decreases with the employment shift towards stagnant activities. For the employment rate, no ambiguity arises: with dominant Baumol (Lewis) dynamics, e decreases (increases) if λ increases. Inequality 4.11 shows that the response of the natural rate of growth is also unambigious; it also declines with λ. However, and as in the two-dimensional version, the decline of g^* is more pronounced if labor markets are Lewisian due to the fall in the labor share.

Inequality 4.10 summarizes the response of the labor share to structural change. Crucially, the labor share always increases following a shift towards stagnant services if Baumol dynamics dominate. In contrast, dominant Lewis dynamics allow for a decline in the labor share in response to the increase in λ. The additional condition is that ω_e is weak, relative to h_e. In other words, capitalists restrict output growth to limit Kaleckian labor militancy, rather than workers exerting power to enact real wage growth. The inequality is also more likely to be satisfied if Lewis dynamics dominate strongly, i.e. if the real wage effects clearly overpower the productivity effect.

5 Concluding remarks

In this paper, we propose a classical-Keynesian theory that links structural change, secular stagnation and a falling labor share in steady state, while maintaining neo-Goodwinian short run mechanisms. To maintain tractability, formal models stick to a one sector representation, and introduce structural change through an exogenous parameter: the employment share of stagnant activities, which are defined as exhibiting no or low labor productivity growth potential.

Further, we define an effect from structural change on growth rates of labor productivity and real wage. If the former outweighs the latter, Baumol dynamics dominate (and vice versa). A key result is that the labor share decreases in steady state in response to a rise in stagnant sector employment shares only (i) in the Keynes-Kalecki version—which overcomes the invariance of the distribution of income to bargaining parameters in the classical version—and if additionally (ii) Lewis dynamics prevail and (iii) the profit squeeze is relatively weak.

We consider the latter two conditions as plausibly satisfied during recent decades, which have been scarred by deregulation, union busting, dismantling of the welfare state, contractionary bias in policy making, a dearth of public investments, increased import competition, and recurring financial crises that undermine steady income and wealth generation for large proportions of the work force. In combination, these factors deprive labor of job opportunities with decent productivity and pay, and undermine labor's bargaining power in remaining progressive activities.

The term *maturity and stagnation* thus could take on new meaning. Steindl (1952) has generally been interpreted to pertain to tendencies of product market concentration with excessive profit margins that lead to consumption-driven realization crises. The perspective adopted here puts labor markets (and related policy making) center stage. Tertiarization and a concomitant slowdown in natural growth might be expected in maturing economies, but dramatic failures in policy making are likely responsible for Lewis dynamics to be dominant, and for the profit squeeze to weaken to the extent that it has. In conclusion, we see structural change as an additional, complementary and complexly interwoven cause of stagnation and inequality. Future research could provide further insights on specific linkages from policy and institutions to the mechanisms underlying this theory.

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A Appendices

A.1 Comparative dynamics: 2D model

We utilize Cramer's rule, and denote $|J^{x,\lambda}|$ as the determinant in the numerator to calculate the partial for variable x w.r.t. λ. Recall that $|J^*| > 0$.

Equation 3.6.

$$\frac{\partial e^*}{\partial \lambda} = \frac{|J^{e,\lambda}|}{|J^*|} = \frac{\begin{vmatrix} ea_\lambda & -e(s_\pi \sigma + a_\psi) \\ \psi(a_\lambda - \omega_\lambda) & -\psi a_\psi \end{vmatrix}}{|J^*|} = \frac{\begin{vmatrix} - & - \\ \psi(a_\lambda - \omega_\lambda) & - \end{vmatrix}}{|J^*|}. \quad \text{(A.1)}$$

Therefore,

$$\frac{\partial e^*}{\partial \lambda} > 0 \Leftrightarrow a_\lambda - \omega_\lambda > 0 \quad \text{(A.2)}$$

$$< 0 \Leftrightarrow a_\lambda - \omega_\lambda < 0 \text{ and } \frac{a_\lambda - \omega_\lambda}{a_\lambda} > \frac{a_\psi}{s_\pi \sigma + a_\psi}. \quad \text{(A.3)}$$

Equation 3.7.

$$\frac{\partial \psi^*}{\partial \lambda} = \frac{|J^{\psi,\lambda}|}{|J^*|} = \frac{\begin{vmatrix} 0 & ea_\lambda \\ \psi\omega_e & \psi(a_\lambda - \omega_\lambda) \end{vmatrix}}{|J^*|}$$

$$= -\frac{ea_\lambda \psi \omega_e}{e(s_\pi \sigma + a_\psi)\psi\omega_e} = -\frac{a_\lambda}{s_\pi \sigma + a_\psi} > 0. \quad \text{(A.4)}$$

Equation 3.8. Given 3.7,

$$\frac{\partial g^*}{\partial \lambda} = a_\lambda \left(\frac{a_\psi}{a_\lambda} \frac{\partial \psi^*}{\partial \lambda} + 1 \right) = a_\lambda \left(\frac{a_\psi}{a_\lambda} \frac{-a_\lambda}{s_\pi \sigma + a_\psi} + 1 \right)$$

$$= a_\lambda \left(-\frac{a_\psi}{s_\pi \sigma + a_\psi} + 1 \right) < 0. \quad \text{(A.5)}$$

A.2 Stability: 3D model

The Routh-Hurwitz conditions for stability of a linear(ized) three-dimensional system of differential equations are

$$Tr(J) < 0 \quad \text{(A.6)}$$

$$|J_{11}| + |J_{22}| + |J_{33}| > 0 \quad \text{(A.7)}$$

$$|J| < 0 \quad \text{(A.8)}$$

$$-Tr(J)(|J_{11}| + |J_{22}| + |J_{33}|) + |J| > 0, \quad \text{(A.9)}$$

The first two are easily verified. Cofactor expansion along the first column determines A.8: $|J| = j_{11}|J_{11}| - j_{21}|J_{21}| < 0$, which is always negative given the signed Jacobian. The fourth inequality (A.9) can be rearranged to give $(j_{11} - Tr(J))|J_{11}| - j_{21}|J_{22}| - Tr(J)(|J_{22}| + |J_{33}|) > 0$, where $j_{11} - Tr(J) = -(j_{22} + j_{33}) = -Tr(J_{11})$. Substituting and distributing gives:

$$\underbrace{-Tr(J_{11})|J_{11}|}_{I}\underbrace{-j_{21}|J_{22}|}_{II}\underbrace{-Tr(J)|J_{22}|}_{III}\underbrace{-Tr(J)|J_{33}|}_{IV} > 0 \tag{A.10}$$

These terms I–IV in turn can be signed:

$$\underbrace{-Tr(J_{11})|J_{11}|}_{\text{I}} = \underbrace{Tr(J_{11})}_{-}e\psi[\underbrace{h_e a_\psi}_{-} + \underbrace{\omega_e(h_\psi - a_\psi)}_{-}] > 0 \tag{A.11}$$

$$\underbrace{-j_{21}|J_{21}|}_{\text{II}} = e\psi u \underbrace{h_u}_{+}[\underbrace{h_e a_\psi}_{-} + \underbrace{\omega_e(h_\psi + s_\pi(1-\psi))}_{-}] < 0 \tag{A.12}$$

$$\underbrace{-Tr(J_{11})|J_{22}|}_{\text{III}} = \underbrace{Tr(J_{11})}_{-}[u\psi\underbrace{(h_u - s_\pi(1-\psi))}_{-}\underbrace{a_\psi}_{+}] > 0 \tag{A.13}$$

$$\underbrace{-Tr(J_{11})|J_{33}|}_{\text{IV}} = \underbrace{-Tr(J_{11})}_{+}[ue(\underbrace{(h_u - s_\pi(1-\psi))h_e}_{+}\underbrace{-h_u h_e}_{+})] > 0 \tag{A.14}$$

A sufficient condition for A.9 to hold is $I + II > 0$. Rearranging gives:

$$\underbrace{[Tr(J_{11}) + uh_u]}_{+/-}\underbrace{h_e a_\psi}_{-} + \underbrace{\omega_e}_{+}[\underbrace{(Tr(J_{11}) + uh_u)}_{+/-}\underbrace{h_\psi}_{-} - \underbrace{Tr(J_{11})a_\psi}_{-} + \underbrace{uh_u s_\pi u}_{+}]$$

which is positive if

$$Tr(J_{11}) + uh_u < 0 \Leftrightarrow -(eh_e - \psi a_\psi) > uh_u. \tag{A.15}$$

A.3 Comparative dynamics: 3D model

See A.1 for remarks on notation. Recall that inequality A.8 holds: $|J^*| < 0$.

Equation 4.8: $\partial u^*/\partial\lambda$.
The determinant in the numerator is

$$|J^{u,\lambda}| = \begin{vmatrix} 0 & uh_e & u(h_\psi + s_\pi u) \\ ea_\lambda & eh_e & eh_\psi \\ \psi(a_\lambda - \omega_\lambda) & \psi\omega_e & -\psi a_\psi \end{vmatrix}. \tag{A.16}$$

If $|J^{u,\lambda}| < 0$, $\partial u^*/\partial\lambda > 0$. Expanding and simplifying the above gives:

$$|J^{u,\lambda}| = e\psi u[a_\lambda\omega_e(h_\psi + s_\pi u) - s_\pi u h_e(a_\lambda - \omega_\lambda) + h_e\omega_\lambda a_\psi]. \tag{A.17}$$

We distinguish the two relevant cases:

1. Lewis effect dominates; $a_\lambda - \omega_\lambda > 0$: The term in the square brackets in (A.17) is positive since $a_\lambda < 0, \omega_\lambda, h_e < 0$ and the economy is profit-led ($h_\psi + s_\pi u < 0$). It follows that $\partial u^*/\partial\lambda < 0$.

2. Baumol effect dominates; $a_\lambda - \omega_\lambda < 0$:

$$\begin{aligned} |J^{u,\lambda}| > 0 &\Leftrightarrow a_\lambda \omega_e (h_\psi + s_\pi u) - h_e(a_\lambda - \omega_\lambda) s_\pi u + h_e \omega_\lambda a_\psi > 0 \\ &\Leftrightarrow a_\lambda \omega_e (h_\psi + s_\pi u) + h_e \omega_\lambda a_\psi > h_e (a_\lambda - \omega_\lambda) s_\pi u > 0 \\ &\Leftrightarrow \frac{\omega_\lambda a_\psi}{s_\pi u a_\lambda} + \frac{\omega_e}{h_e}\left(1 + \frac{h_\psi}{s_\pi u}\right) > \frac{(a_\lambda - \omega_\lambda)}{a_\lambda}. \end{aligned} \tag{A.18}$$

If inequality (A.18) holds, $\partial u^*/\partial\lambda < 0$.

Equation 4.9: $\partial e^*/\partial\lambda$.
The determinant in the numerator is

$$|J^{e,\lambda}| = \begin{vmatrix} u(h_u - s_\pi(1-\psi)) & 0 & u(h_\psi + s_\pi u) \\ e h_u & e a_\lambda & e(h_\psi - a_\psi) \\ 0 & \psi(a_\lambda - \omega_\lambda) & -\psi a_\psi \end{vmatrix}. \tag{A.19}$$

If $|J^{e,\lambda}| < 0$, $\partial e^*/\partial\lambda > 0$. Expanding the above gives:

$$\begin{aligned} |J^{e,\lambda}| &= u[h_u - s_\pi(1-\psi)][-e a_\lambda \psi a_\psi - e\psi(h_\psi - a_\psi)(a_\lambda - \omega_\lambda)] \\ &\quad + e h_u [u(h_\psi + s_\pi u)\psi(a_\lambda - \omega_\lambda)] \\ &= eu\psi[(h_u - s_\pi(1-\psi))(-h_\psi(a_\lambda - \omega_\lambda) - a_\psi \omega_\lambda) \\ &\quad + h_u(h_\psi + s_\pi u)(a_\lambda - \omega_\lambda)]. \end{aligned} \tag{A.20}$$

The two cases are:

1. Lewis effect dominates; $a_\lambda - \omega_\lambda > 0$. A.20 is clearly negative.

2. Baumol effect dominates; $a_\lambda - \omega_\lambda < 0$: A.20 is always positive, since $-a_\lambda a_\psi - (h_\psi - a_\psi)(a_\lambda - \omega_\lambda) < 0 \Leftrightarrow -h_\psi(a_\lambda - \omega_\lambda) - \omega_\lambda a_\psi < 0 \Leftrightarrow \frac{a_\lambda - \omega_\lambda}{\omega_\lambda} > \frac{a_\psi}{h_\psi} > 0$.

Equation 4.10: $\partial\psi^*/\partial\lambda$.
The determinant in the numerator is

$$|J^{\psi,\lambda}| = \begin{vmatrix} u(h_u - s_\pi(1-\psi)) & u h_e & 0 \\ e h_u & e h_e & e a_\lambda \\ 0 & \psi\omega_e & \psi(a_\lambda - \omega_\lambda) \end{vmatrix}. \tag{A.21}$$

If $|J^{\psi,\lambda}| < 0$, $\partial\psi^*/\partial\lambda > 0$. Expanding the above gives:

$$|J^{\psi,\lambda}| = j_{11}|J^{\psi,\lambda}_{11}| - j_{21}|J^{\psi,\lambda}_{21}|, \tag{A.22}$$

where $j_{11} < 0$, $j_{21} > 0$, but signs of minors depend on Baumol and Lewis effects:

1. Lewis effect dominates; $a_\lambda - \omega_\lambda > 0$: First, the determinant simplifies to

$$|J^{\psi,\lambda}| = -e\psi u[s_\pi(1-\psi)[h_e(a_\lambda - \omega_\lambda) - a_\lambda\omega_e] + a_\lambda\omega_e h_u]. \tag{A.23}$$

A sufficient condition for (A.22) to be positive so that $\frac{\partial\psi^*}{\partial\lambda} = \frac{|J^{\psi,\lambda}|}{|J^*|} < 0$ is that $|J_{11}^{\psi,\lambda}| < 0$. This sufficient condition is

$$e\psi(h_e(a_\lambda - \omega_\lambda) - \omega_e a_\lambda) < 0 \Leftrightarrow \frac{a_\lambda - \omega_\lambda}{a_\lambda} < \frac{\omega_e}{h_e} < 0. \tag{A.24}$$

If instead $\frac{a_\lambda - \omega_\lambda}{a_\lambda} > \frac{\omega_e}{h_e}$, either case can occur. We work now with the expanded form of $|J^{\psi,\lambda}|$. Assume that the term in the square bracket is negative which would imply that $\frac{\partial\psi^*}{\partial\lambda} < 0$ and the labor share responds negatively to structural change.

$$\begin{aligned}
|J^{\psi,\lambda}| > 0 &\Leftrightarrow a_\lambda\omega_e(h_u - s_\pi(1-\psi)) + s_\pi(1-\psi)h_e(a_\lambda - \omega_\lambda) < 0 \\
&\Leftrightarrow a_\lambda\omega_e(h_u - s_\pi(1-\psi)) < -s_\pi(1-\psi)h_e(a_\lambda - \omega_\lambda) \\
&\Leftrightarrow \frac{\omega_e}{h_e}(h_u - s_\pi(1-\psi)) < -s_\pi(1-\psi)\frac{(a_\lambda - \omega_\lambda)}{a_\lambda} \\
&\Leftrightarrow \frac{\omega_e}{h_e}(1 - \frac{h_u}{s_\pi(1-\psi)}) > \frac{(a_\lambda - \omega_\lambda)}{a_\lambda}
\end{aligned} \tag{A.25}$$

Given $\frac{a_\lambda - \omega_\lambda}{a_\lambda} > \frac{\omega_e}{h_e}$, and that both terms are negative, the inequality (A.25) is satisfied and $\frac{\partial\psi^*}{\partial\lambda} < 0$ if the term $(1 - \frac{h_u}{s_\pi(1-\psi)}) \in (0,1)$ is closer to zero rather than 1 such that the negative ratio ω_e/h_e is reduced. The economic intuition is that the partial effects of demand (u) on the actual growth rate h and on warranted growth rate g must be close in magnitude. If, on the other hand, the warranted growth rate responds more strongly to u the inequality is reversed and $\frac{\partial\psi^*}{\partial\lambda} > 0$ as in Baumol's case.

2. Baumol effect dominates; $a_\lambda - \omega_\lambda < 0$: $|J_{11}^{\psi,\lambda}| > 0, |J_{21}^{\psi,\lambda}| > 0 \Leftrightarrow |J^{\psi,\lambda}| \Leftrightarrow \partial\psi^*/\partial\lambda > 0$.

Equation 4.11.
The natural rate of growth's response to changes in economic structure evolves according to equation 2.7. Since $a_\lambda < 0$ and $a_\psi > 0$ it follows immediately that g^* declines if the labor share share responds negatively to λ, i.e. $\frac{\partial\psi^*}{\partial\lambda} < 0$. This can happen under the Lewis case only as derived above. It remains therefore to check if the natural rate of growth can exhibit a different behavior in those cases when the labor share rises with λ. Before we derive relevant conditions recall that:

$$\frac{\partial\psi^*}{\partial\lambda} = \frac{|J^{\psi,\lambda}|}{|J^*|}$$

$$= \frac{-e\psi u[s_\pi(1-\psi)[h_e(a_\lambda - \omega_\lambda) - a_\lambda \omega_e] + a_\lambda \omega_e h_u]}{e\psi u[s_\pi(1-\psi)(h_e a_\psi + \omega_e(h_\psi - a_\psi)) + h_u\omega_e(s_\pi u + a_\psi)]}, \text{ and therefore}$$

$$\frac{\partial g^*}{\partial \lambda} = a_\lambda \left(\frac{a_\psi}{a_\lambda} \frac{\partial \psi^*}{\partial \lambda} + 1 \right) = a_\lambda \left(\frac{a_\psi}{a_\lambda} \frac{|J^{\psi,\lambda}|}{|J^*|} + 1 \right)$$

$$= a_\lambda \left(\frac{-[s_\pi(1-\psi)(h_e \frac{a_\lambda - \omega_\lambda}{a_\lambda} - \omega_e) + \omega_e h_u]}{s_\pi(1-\psi)[h_e + \omega_e(\frac{h_\psi}{a_\psi} - 1)] + \omega_e h_u(\frac{s_\pi u}{a_\psi} + 1)} + 1 \right) \tag{A.26}$$

where $|J^{\psi,\lambda}|, |J^*| < 0$ and hence $\frac{\partial \psi^*}{\partial \lambda} > 0$. The fraction in (A.26) is negative to begin with—we consider Baumol case and Lewis case when condition (A.25) does *not* hold, and $\frac{a_\lambda - \omega_\lambda}{a_\lambda} > \frac{\omega_e}{h_e}$). For $\frac{\partial g^*}{\partial \lambda} > 0$ this fraction must be less than -1[16] or larger in absolute terms larger than 1. This implies:

$$s_\pi(1-\psi)(h_e \frac{a_\lambda - \omega_\lambda}{a_\lambda} - \omega_e) + \omega_e h_u < s_\pi(1-\psi)[h_e + \omega_e(\frac{h_\psi}{a_\psi} - 1)] + \omega_e h_u(\frac{s_\pi u}{a_\psi} + 1)$$

$$h_e \frac{a_\lambda - \omega_\lambda}{a_\lambda} < h_e + \omega_e \frac{h_\psi}{a_\psi} + \frac{\omega_e h_u u}{a_\psi(1-\psi)}$$

$$h_e \frac{a_\lambda - \omega_\lambda}{a_\lambda} < h_e + \frac{\omega_e}{a_\psi}(h_\psi + \frac{h_u u}{1-\psi}) \tag{A.27}$$

We now consider the two relevant cases:

1. Lewis effect dominates; $a_\lambda - \omega_\lambda > 0$, additionally with $\frac{a_\lambda - \omega_\lambda}{a_\lambda} < 0$. Give that $h_e < 0$ the left hand side term of the inequality (A.27) is always positive. If the second term on the right hand side is negative, the right hand side is negative and (A.27) can not hold. It follows that $\frac{\partial \psi^*}{\partial \lambda} > 0$ can never be true under the Lewis case. Specifically, we must prove that $h_\psi + \frac{h_u u}{1-\psi} < 0$ since $\frac{\omega_e}{a_\psi} > 0$. (Indeed, this is the case: $a_{11} = h_u - (1-\psi)s_\pi < 0$ in J captures the Keynesian stability condition; and $a_{13} = h_\psi + s_\pi u < 0$ captures the profit-led character of the economy. Multiplying a_{11} by u, a_{13} by $(1-\psi)$ and adding them proves that $h_\psi(1-\psi) + h_u u < 0$.)

2. Baumol effect dominates; $a_\lambda - \omega_\lambda < 0$, additionally with $0 < \frac{a_\lambda - \omega_\lambda}{a_\lambda} < 1$. Since h_e on the left hand side is multiplied by a number between $(0, 1)$ it follows that the inequality, again, cannot hold given that $h_e \frac{a_\lambda - \omega_\lambda}{a_\lambda} > h_e$. Hence, even in Baumol's case we have that $\frac{\partial \psi^*}{\partial \lambda} < 0$ despite the clear rise in ψ.

[16]This basically translates into $\frac{a_\psi}{a_\lambda} \frac{\partial \psi^*}{\partial \lambda} + 1 < 0$ and hence $\frac{\partial g^*}{\partial \lambda} > 0$ given that $a_\lambda < 0$.