DEPARTMENT OF ECONOMICS WORKING PAPER SERIES

# Advertising Economics Under Uncertainty: An Alternative Approach

James P. Gander

**Working Paper No: 2018-01**

January 2018

University of Utah
Department of Economics
260 S. Central Campus Dr., Rm. 343
Tel: (801) 581-7481
Fax: (801) 585-5649
http://www.econ.utah.edu

# Advertising Economics Under Uncertainty: An Alternative Approach

**James P. Gander**

**Department of Economics, University of Utah, Salt Lake City, Utah, 84112**

**gander@economics.utah.edu**

## Abstract

**The purpose of this paper is to present an alternative approach to analyzing firm advertising under uncertainty. The approach considers the simultaneity (or duality) of two effects of advertising, one effect on the probability associated with the bundle of goods the typical buyer purchases and the other effect on the probability associated with the time the buyer spends in the store making the purchases. While bundle and time are well explored in the literature, our simultaneity approach to determine the optimum level (and type) of advertising results in implications that are not present in the literature. The novelty of this alternative approach is that it shows that there can be the possibility of an equivalent dual optimal advertising effect on the expected value of the bundle and the expected value of the time spent. The implications of such an equivalence (or lack thereof) for advertising decision making are then explored.**

**Keywords: Advertising, Uncertainty, Bundle, Time Spent, Equivalence**
**JEL Classification: 022, 024, 511, 541**

# Advertising Economics Under Uncertainty: An Alternative Approach

By James P. Gander

## 1. Introduction

Traditionally, based on the work by Stigler and Becker (1977), Dixit and Norman (1978), Fisher and McGowan (1979), Nichols (1985), Hochman and Luski (1988), Becker and Murphy (1993), and Bagwell (2005) and others, firm advertising has been analyzed by placing it explicitly into the typical consumer's utility function (cardinal or ordinal, though not clear which). The focus of the analysis has been on the welfare effects of advertising, where welfare is defined as consumers' surplus plus profits. The effect is welfare increasing or decreasing depending on the market structure of the firm(s) supplying the product in question and its corresponding advertising (free to the buyer or for a price).

Recently, with the advent of on-line shopping, the focus of advertising has been expanded to include entertainment and enjoyment aspects of shopping and their hedonic or utility effects (See, Childers, et al., 2001).

In terms of the pure theory of advertising, the approach has usually been dynamic along the lines of capital theory (see, for example, Arrow and Nerlove, 1962). Advertising, like capital investment, adds to the firm's capital stock of "Goodwill." The focus, then, of the dynamic approach is to specify the optimal time path of advertising. This approach has also been expanded to include stochastic (random) dynamic control models (See, Du, Hu, and Ai, 2007; Nguyen, 1985; and Aykac, et al., 1989).

A key aspect of the advertising literature, then, has been to recognize its uncertain effect on consumer spending. For our purpose, we will focus our literature discussion on the static approach rather than on the more complicated dynamic approach. With this in mind, the static literature has approached uncertainty from two directions or sources, one by specifying a known buyer's response function (dollar value of goods purchased) to advertising subject to a random disturbance (see, for example, Nguyen, 1985) and another by recognizing that the firm's empirically estimated buyer-response function is itself uncertain. Because of this latter source of uncertainty, Aykac, et al. (1989) argued that the principle use of advertising is to reduce the variability of profit and consumer sales (in other words the risk facing the firm).

Also recognized in the literature are two key aspects of buyer purchases, the bundle (in terms of its width and depth) of goods purchased and the related aspect of the time a typical buyer spends in the store making the purchase (See, for a discussion of bundle composition and buyer response, Harlam, et al., 1995 and for time spent in the store, see, Park, et al., 1989 and Barli, et al., 2012).

Our focus here is also on uncertainty and its effect on the bundle of goods purchased and on the time spent in the store. The novelty of our focus is that we recognize the dual effect of advertising on both the probability effect of advertising on the dollar value of the bundle and its probability effect on the time spent in the store. As a result of these dual or simultaneous effects there is a possible equivalence between the value of the bundle and the value of time spent in the store. Our argument is that this duality affects the advertising optimization process. Our main contribution is the implications of this duality effect on the type of advertising to pursue.

To summarize briefly, our basic position (consistent with the literature) is that firms are not particularly interested in the specific product a consumer buys, but in the bundle of products purchased at a given time. For example, we suspect that Wal-Mart is interested in the total dollar expenditure on the whole bundle of goods the typical consumer buys at a given time. In other words, what counts to the firm is the dollar value of a bundle of products. The purpose of advertising is to increase the value of the bundle by getting the buyer to widen the bundle (in terms of more items like food, clothing, house wares, jewelry, toys, etc.) and to deepen the bundle (in terms of more quantity of each item). Under uncertainty, the purpose of advertising is to influence the probability of the dollar value of the bundle.

For another example, the same may be said of the interest of the typical automobile dealer. The VW Passat dealer is interested in the buyer's total expenditure on the total bundle purchased (the body type, safety components, electronics, seat covering, engine type, tires, and the list goes on). Advertising of various types (information, persuasion, snob appeal, and others) is used to increase the dollar value of the bundle. Under uncertainty, it is the dollar value of the bundle that is uncertain in a probabilistic sense. This is our position in designing an alternative approach to analyzing advertising.

Related to the "depth"/"width" structure of the buyer's bundle, as indicated earlier, is the length of time the typical buyer spends in the store. Our alternative approach considers all three of these dimensions (with emphasis on depth) and recognizes that advertising has, in effect, a dual role, affecting both the dollar value of the typical buyer's bundle and the time spent in the store. As a result of the dual effect, we consider that there could be an equivalent result such that the dollar value of the bundle will be equivalent to the dollar value of the typical buyer's time spent in the store. The importance of such equivalence to the design of advertising will be considered later.

In what follows, in the next section the basic elements of the alternative approach are given and uncertainty is introduced into the approach. In the subsections, the direct and indirect approaches with corresponding probabilities under uncertainty are shown. Then, the next section contains the advertising optimization procedure. The next section considers the simultaneous dual effect of advertising on the probabilities associated with the bundle and time spent. The final section contains a summary and conclusions.

## 2. An Alternative Approach

For what follows, we define the total expenditure (EX) by the typical consumer on a given bundle using vector notation as EX(A) = P1X1 + P2X2 + … + PnXn = PX(A), where A is advertising and the P's are the given product prices. The width (given by n) and depth (given by the quantity-value of the X's) of the bundle are the outcomes of the consumer's response to the amount and type of a given A.

We also define an equivalent expression that reflects the time a buyer spends in the store shopping for the bundle (ignoring other time spent). The time period will be affected by the depth and width of the bundle. The time-period expression is given by T(A) = t1X1 + t2X2 + … + tnXn = TX(A) in vector notation, where the $t_i$'s are the time-prices. Let "r" be the average money spent per unit of time (say, in dollars per minute), then under equivalence, EX(A) = rTX(A). In effect, the more time the typical buyer spends in the store, the more will be spent on the bundle

Under uncertainty, however, equivalency may not hold, because of the way advertising affects the probabilities associated with the bundle and time spent in the store. From the firm's perspective, the buyer's choice solution EX(A) is not known with certainty, nor is the buyer's TX(A) time spent shopping known with certainty. In consideration of our duality argument, each of these uncertainties has its own corresponding probabilities, each of which is dependent on advertising expenditures, A. With these equivalent (possibly) expressions in mind, there are two ways to approach the advertising optimization problem, direct and indirect. In what follows, we consider both ways.

## 2.1 The Direct Approach

With the direct approach, the firm estimates the EX(A)'s for any given A. The firm may produce an estimated frequency distribution of the typical buyer's EX(A)'s for different amounts of A. For simplicity of model design, we can assume that the firm estimates only two levels of bundle expenditures for any A, a high level, $EX_{high}$, with a probability of p(A) and a low level, $EX_{low}$, with a probability of (1 – p(A)). The expected revenue for the optimization problem is given by $ER(A) = p(A)EX_{high} + (1 – p(A))EX_{low}$. In other words, we take the estimates of possible outcomes as given with the corresponding probabilities.

As indicated earlier, there is a simultaneity problem here for A can affect both the bundle and the time probabilities. We will address this problem shortly.

## 2.2 The Indirect Approach

With the indirect approach, going back to the buyer's time expression, we assume two TX(A)'s estimates, $TX(A)_{high}$ and $TX(A)_{low}$ that correspond to the two probabilities, p^(A) and (1 - P^ (A)). Similar to before, the firm's expected buyer's time period is given by $ETX(A) = p\^{} (A)TX(A)_{high} + (1 – p\^{} (A))TX(A)_{low}$ . The firm can make the two time estimates by clocking the entry and exit of buyers to the store, using suitable electronic devices.

To summarize briefly, we recognize two sources of uncertainty, time spent and money spent by the buyer. Both corresponding probabilities are jointly affected by A. But, as we will show, the effects need not be the same and the difference in the effects will have an impact on the optimization procedure.

## 3. The Optimization Procedure

The alternative approach under uncertainty focuses on how the firm chooses its optimal level of advertising, A*, given its two probability functions, p(A).and p^(A). This is the optimization problem. As indicated earlier, we consider two equivalent approaches to this problem. In the direct approach, as shown before, the firm's expected revenue from the two possible bundles (high and low) is given by $ER(A) = p(A)EX_{high} + (1 – p(A))EX_{low}$ . The firm's total cost is given by C(A, X) = C(X) + aA, where for simplicity we assume that C(X) = 0. Also, with "a" we ignore any possible economies of scale or of scope. Expected profit is given by, Ep(A) = ER(A) – aA. As before, we focus on the typical buyer, taken as given the actual number of buyers in the store at any given period of time. To simplify the optimization problem, we take as given the high/low estimates as indicated earlier. The firm's optimal A* will then satisfy the first-order condition, $\partial p(A)/\partial A = a/(EX_{high} - EX_{low})$, a positive constant by design.

Due to the equivalence principle we are analyzing, the optimization problem for determining the optimal A* by the indirect approach will have a similar first-order condition given by $\partial p\hat{}(A)/\partial A = a/r(ETX_{high} - ETX_{low})$, based on expected profit now given by $Ep(A) = r[p\hat{}(A)TX(A)_{high} + (1 - p\hat{}(A))TX(A)_{low}] - aA$, where "r" is the average time price as given earlier.

4. Simultaneity Problem

As indicated earlier, spending on A affects both probabilities p(A) and p^(A), jointly. If equivalency holds, then the expected profit from the bundle and the time period will be the same. In other words, spending on A has the same dual effect (like feeding the lamb produces both wool and mutton of the same value). With equivalency, the firm will be indifferent as to the nature or type of advertisement produced, one that increases the depth of the bundle versus one that increases the time spent in the store.

But, if p and p^ react differently to an increase in A, the firm will have a preference as to the type of advertisement to produce. For example, say p(A) rises faster than p^(A), as they both approach the probability limit of 1 (for illustration purposes, let $p(A) = aA^b$, where $0 < a < 1$ and $0 < b < 1$, say .63 and .10 respectively and similarly for p^(A) but with $b^{\wedge} = .05$). Then, the firm will expect more profit from the depth of the bundle (given its width, n) than from the time period. We can hypothesize that such a relative profit gain will affect the nature or type of advertisement, so the focus of A will be on the depth of the bundle (by providing, for example, larger carts for more quantities) rather than on the time period spent in the store, per se.

On the other hand, the reverse may be the case, where p^(A) rises faster than p(A) as A increases. In this situation, the firm will want to keep buyers in the store longer by various means (music, side shows, product demonstrations, product sampling, and the like).

Of course, if spending on A equally affects both p and p^, then the equivalency argument holds and the firm is indifferent as to the type of A, as discussed before. The problem with indifference is that the firm's advertisement manager can face a decision dilemma and indeterminacy. How such a dilemma is resolved in theory and in practice is beyond the scope of this paper, suffice it to say that more information may be needed in order to resolve the dilemma. We leave this dilemma open for future discussions.

## 5. Summary and Conclusions

An advertising model was designed based on the typical buyer's depth and width of his/her bundle of products purchased. Also recognized was the time spent in the store purchasing the bundle. Uncertainty was introduced into the model, where two probabilities, one for the bundle and one for the time period, were defined. A duality was recognized in that both probabilities were dependent on advertising spending, A. This duality or simultaneity could result in the equivalence between the dollar value of the bundle and the dollar value of the buyer's time spent in the store. With such a equivalence, a decision problem may occur. If equivalence does not exist, then the firm will under optimization select the best (in terms of expected profit) form or type of advertisement program to follow, one that favors either the depth of the bundle or the length of time spent in the store.

Our equivalence approach has added to the literature on advertising. The multi-role nature of advertising makes it a very complex phenomenon to fully understand, involving not just finance, management science, and economics, but also psychology. We hope that our equivalence approach has contributed something to a wider understanding of this complex phenomenon.

References

Arrow, K. J. and Nerlove, M. 1962. Optimal advertising policy under dynamic

Conditions. Economica 29(113) 129-142.

Aykac, A., M. Corstjens, D. Gautschi and I. Horowitz. 1989. Estimation Uncertainty and Optimal Advertising Decisions. Management Science 35(1) 42-50.

Bagwell, K. 2005. The Economic Analysis of Advertising. Columbia University Department of Economics, Discussion Paper Series, No.: 0506-01, August, 1-181.

Barli, Ö., M. Aktan, B. Bilgili, and S. Dane. 2012. Lighting, indoor color, buying behavior and time spent in a store. Color Research & Application 37(6) 465-468.

Becker, G. S., K. M. Murphy. 1993. A Simple Theory of Advertising as a Good or Bad. Quarterly Journal of Economics 108(4) 941-964.

Childers, T.L.,L.C. Christopher, J. Peck, and S. Carson. 2001. Hedonic and Utilitarian motivations for online retail shopping behavior. Journal of Retailing 77(4) 511-535.

Dixit, A., V. Norman. 1978. Advertising and Welfare. Bell Journal of Economics 9 (1) 1-17.

Du, R., Q. Hu, and S. Ai. 2007. Stochastic optimal budget decision for advertising Considering uncertain sales responses. European Journal of Operational Research 183 1042-1054.

Fisher, F. M., J. J. McGowan. 1979. Advertising and Welfare: Comment, Bell Journal of

Economics 10(2) 726-27.

Harlam, B. A., A. Krishna, D. R. Lehmann, and C. Mela. 1995. Impact of bundle type, price framing and familiarity on purchase intention for the bundle. Journal of Business Research 33(1) 57-66.

Hochman, O., I. Luski. 1988. Advertising and Economic Welfare: Comment. American Economic Review 788(1) 290-96.

Nguyen, D. 1985. An Analysis of Optimal Advertising under Uncertainty Management Science 31(5) 622-633.

Nichols, L. M. 1985. Advertising and Economic Welfare. American Economic Review 75(1) 213-18.

Park, C. W., E. S. Lyer, and D. C. Smith. 1989. The Effects of Situational Factors on In-Store Grocery Shopping Behavior: The Role of Store Environment and Time Available for Shopping. Journal of Consumer Research 15(4) 422-433.

Stigler, G. J., G. S. Becker. 1977. De Gustibus Non Est Disputandum, [There is no disputing about tastes]. American Economic Review 67(2) 76-90.