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Is Potential Output Growth Falling?

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Abstract

We document differences between the evolution of a measure of potential output growth and the evolution of a measure of potential output per capita growth using time-varying parameter models estimated for four advanced economies (Canada, Germany, the United Kingdom and the United States). The evidence supports the view that most of the slowdown in potential output growth occurred prior to the Great Recession. However, the potential output per capita growth rate: 1) remained relatively constant in Canada; and 2) decreased less (more) than the potential output growth rate in Germany and the United States (in the United Kingdom). These results indicate that: 1) the decline in potential output growth in Canada is mainly associated with the decrease in population growth; and 2) the decrease in population growth is an important factor in order to explain the decline in potential output in Germany and the United States, but not in the United Kingdom.

Keywords: Potential output growth rate, Potential output per capita growth rate, Rates of growth consistent with a constant unemployment rate.
JEL Classification: O41, O47.

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1 Introduction

The Great Recession (GR) has raised concerns about the possibility that advanced economies are entering an era of secular stagnation, that is, an era characterised by a slowdown in the rate of growth of potential GDP. Indeed, the evidence of a decline in long-run growth in advanced economies is accumulating, as documented by the recent growth literature (Antolin-Diaz et al. , 2016; Benati , 2007; ECB , 2011; Fernald , 2007; 2014; Fernald and Wang , 2015; Gordon , 2012; 2013; 2014a;b; 2015).

As Basu and Fernald (2009: 205) explain, "[e]stimating potential output growth is one modest example and relatively transparent example of [the] interplay between theory and measurement"

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and, therefore, its estimation is surrounded by considerable uncertainty since the latter reflects the ongoing controversy regarding the origins of economic fluctuations (Cerra and Saxena , 2000). This paper compares the evolution of a measure of potential output growth —the rate of growth of output that keeps the unemployment rate constant— with the evolution of a measure of potential output per capita growth —the rate of growth of output per capita that keeps the unemployment rate constant— in four advanced economies —Canada, Germany, the United Kingdom (UK) and the United States (US)— during the post-war era. We employ time-varying parameter models and a Heckman-type two step estimation procedure that deals with the possible endogeneity problem, finding differences between both potential growth rates. The evidence supports the view that the potential output growth rate decreased in the four countries of study prior to the GR. However, the potential output per capita growth rate: 1) remained relatively constant in Canada; 2) decreased in Germany, the UK and the US; and 3) decreased less (more) than the potential output growth rate in Germany and the US (in the UK). These results indicate that: 1) the decline in potential output growth in Canada is mainly associated with the decrease in population growth; and 2) the decrease in population growth seems to be an important element in order to explain the decline in potential output in Germany and the US, but not in the UK.

Our research is, firstly, related to recent studies that have documented substantial fluctuations over time in output growth because of a variety of mechanisms that can affect its evolution, such as highly persistent negative effects derived from major economic crisis. For a large set of 190 countries, Cerra and Saxena (2008) find that less than 1% point of the deepest output loss is regained by the end of ten years following various financial and political crisis; and that the magnitude of persistent output loss ranges from around 4% to 16% for the shocks analysed. Galí et al. (2012) study the performance of GDP, employment and other labour market variables following the troughs in postwar US business cycles, finding slower recoveries in the three most recent episodes. Stock and Watson (2012) show that most of the slowness of the recovery in the US after the GR was attributable to the plateau in the female labour force participation rate and to the ageing of the workforce; concluding that future recessions will be deeper and longer, and will have slower recoveries. Using data on 23 advanced economies over the past 40 years, Martin et al. (2015) also find little evidence that growth is faster following recessions —if anything post-trough growth is slower. Likewise, Blanchard et al. (2015) use a sample of 23 advanced economies over the last 50 years, finding that: 1) about two-thirds of the recessions are followed by lower output relative to the pre-recession trend —even after the economy has recovered; 2) in about one-half of those cases, the recession is followed not just by lower output, but also by lower output growth; and 3) recessions possibly triggered by demand shocks (intentional disinflations) are often followed by lower output or even lower output growth.

The fact that deep and prolonged recessions can depress the long-run level of output may imply that demand shocks and/or short-run fluctuations have permanent effects on economic growth, thus suggesting the existence of hysteresis effects (DeLong and Summers , 2012; ECB , 2011; León-Ledesma and Thirlwall , 2002): reductions in labour force growth, by discouraging groups in the labour force from participating in the labour market and by reducing immigration flows; reductions in capital investment and in research and development because economic crisis can depress current and expected profits over a protracted period and can lead to increases in risk premia, resulting in tighter lending standards and higher effective borrowing costs; reduced experimentation with business models and information spillovers; changes in managerial attitudes; and weak labour demand and persistent patterns of lower employment growth.

Eggertsson and Mehrotra (2014) have recently developed an overlapping generations model with nominal wage rigidity, showing that any combination of a permanent collateral (deleveraging) shock, slowdown in population growth, or an increase in inequality can lead to a permanent output shortfall.

Finally, our research is also related to the recent growth literature that has documented a reduction in different measures of potential output (Antolin-Diaz et al. , 2016; Benati , 2007; ECB , 2011; Fernald , 2007; 2014; Fernald and Wang , 2015; Gordon , 2012; 2013; 2014a;b; 2015). It is possible to summarise the main findings of these studies as follows. Firstly, the evidence seems to support the view that there has been a gradual decline —rather than a discrete break— in long-run output growth. Secondly, most of the slowdown in potential output occurred prior to the GR. Thirdly, the decline in the growth rate of labour productivity appears to be behind the slowdown in potential output growth in the US. As Benati (2007), Fernald (2007; 2014) and Gordon (2012; 2013; 2014a;b; 2015) show, the evolution of productivity growth in the US can be characterised as follows: high productivity growth in the 1950s and 1960s as a consequence of the inventions derived from the second industrial revolution (airplanes, air conditioning, interstate highways); productivity growth slowed down after 1973; information technology (the third industrial revolution: computers, the web, mobile phones) created only a short-lived productivity growth revival from mid-1990s and early 2000s; and productivity growth slowed again before the GR and it has practically vanished during the past decade.[1] Fourthly, the weakening in labour productivity prior to the GR also appears to be a global phenomenon (Antolin-Diaz et al. , 2016; Cette et al. , 2016). Fifthly, with respect to the Euro-zone, the ECB (2011) also points out that, irrespective of the long-run effects of the financial crisis on potential growth, the ageing population will have a dampening effect on future potential output growth as there has been a reduction in the size of its working age population.

In this sense, it is possible to find the following advantages derived from the alternative empirical approach adopted in this paper. Firstly, we provide estimates of potential output growth rates that cover a longer period compared with the estimates provided by the International Monetary Fund (IMF) —which are available only from 1981. Secondly, to the best of our knowledge, ours is the first paper that explicitly estimates a measure of potential output per capita growth, which allows us to compare the possible differences in the evolution of the latter with the evolution of potential output growth. As the results obtained show, this is of utmost importance because: 1) the evolution of population growth in the countries of study has been substantially different; and 2) the differences in the behaviour of both potential output growth rates cannot be captured adequately using other more conventional methods, such as the Hodrick-Prescott (HP) filter.

The rest of the paper is organised as follows. The following section presents the econometric models and techniques employed. Section 3 presents the main results; and compares them with the potential output growth rates estimated by the IMF and by the Congressional Budget Office (CBO), and with the results obtained from the HP filter. Finally, the main conclusions are presented in section 4.

[1] Byrne et al. (2016) show that there is little evidence that the slowdown in the growth rates in labour productivity and total factor productivity arises from growing mismeasurement of the gains from innovation and information technology-related goods and services.

2 Empirical strategy

2.1 Derivation of the time-varying parameter models

Suppose that output Y is produced via the following aggregate production function with disembodied technological progress[2]:

$$Y = AF(K,N) = AF(kc,nh) \tag{1}$$

where K is the capital input, N is the labour input, k is the number of capital stock, c is the utilization rate, n denotes the number of workers employed, h is the number of working hours, and A is a measure of the ability of the economy to transform inputs into output, that is, the state of technology.

Equation (1) can be expressed in growth rates as follows:

$$g_t = a_t + \alpha\,(k_t + c_t) + \theta\,(n_t + h_t) \tag{2}$$

where g_t, a_t, k_t, c_t, n_t, and h_t denote the rates of growth of Y, A, k, c, n and h, respectively; whereas $\alpha = (\delta F/\delta K)(K/F)$ and $\theta = (\delta F/\delta N)(N/F)$ represent the elasticities of Y with respect to K and to N, respectively.

It is possible to employ the following logarithmic transformation on the unemployment identity: $u_t \approx \ln L - \ln N$, where u_t represents the unemployment rate and L represents the labour force. Therefore, $\Delta u_t \approx l_t - n_t$, where Δu_t denotes the change in the unemployment rate and l_t is the rate of growth of the labour force; and $n_t \approx l_t - \Delta u_t$. Hence, equation (2) can be expressed as follows:

$$g_t = a_t + \alpha\,(k_t + c_t) + \theta\,(l_t - \Delta u_t + h_t) \tag{3}$$

As discussed in the previous section, there is evidence that suggests that the rate of growth of potential output can experience fluctuations over time. We incorporate the latter using equation (3). Firstly, the rate of growth of multifactor productivity (a_t) can be influenced by the actual output growth rate (g_t) because of the learning-by-doing process (Arrow , 1962) and because of the Kaldor-Verdoorn mechanism (León-Ledesma and Thirlwall , 2002): productivity can be considered a function of cumulative output; the more (less) output produced, the more (less) adept labour becomes at producing it. If a_t is a positive function of g_t, then $a_t = \gamma_0(g_t)$, where $\gamma_0 > 0$ measures the sensitivity of a_t with respect to g_t.

Secondly, the literature has also documented that the capacity utilization rate is a pro-cyclical variable, increasing during expansions and decreasing during recessions. Therefore, if c_t can be influenced by g_t, then $c_t = \gamma_1(g_t)$, where $\gamma_1 > 0$ measures the sensitivity of c_t with respect to g_t.

Thirdly, labour force growth (l_t) can also react to g_t because of: 1) the discouraged workers effect: during recessions (as the unemployment rate increases) labour force participation rates fall because workers may simply not look for a job or may give up looking; and during expansions (as the unemployment rate falls) labour force participation rates rise because workers out of the labour force re-enter the labour force; and 2) labour immigration towards booming labour markets. If l_t can react to g_t, then $l_t = \gamma_2(g_t)$, where $\gamma_2 > 0$ quantifies the sensitivity of l_t with respect to g_t.

Fourthly, hours worked can also be affected by g_t because of labour hoarding practices: as training new employees is costly, firms prefer to keep current workers rather than lay them off

[2] This derivation is similar to the one presented by Huang and Lin (2008).

during recessions, and ask them to work over time rather than hire new employees during expansions. Thereby, $h_t = \gamma_3(g_t)$, where $\gamma_3 > 0$ measures the sensitivity of h_t with respect to g_t.

Therefore, substituting $a_t = \gamma_0(g_t)$, $c_t = \gamma_1(g_t)$, $l_t = \gamma_2(g_t)$ and $h_t = \gamma_3(g_t)$ into equation (3):

$$g_t = \gamma_0(g_t) + \alpha\,(k_t + \gamma_1(g_t)) + \theta\,(\gamma_2(g_t) - \Delta u_t + \gamma_3(g_t)) \tag{4}$$

Finally, there is also substantial empirical evidence that shows the presence of an asymmetric behaviour between output and unemployment, so that it is necessary to consider the the possibility that Okun's coefficient for different time points might be dissimilar. We incorporate the time-varying behaviour of the Okun coefficient on unemployment, so that θ_t:

$$g_t = \gamma_0(g_t) + \alpha\,(k_t + \gamma_1(g_t)) + \theta_t\,(\gamma_2(g_t) - \Delta u_t + \gamma_3(g_t)) \tag{5}$$

$$g_t = g_t\,(\gamma_0 + \alpha\gamma_1 + \theta_t\gamma_2 + \theta_t\gamma_3) + \alpha\,(k_t) - \theta_t\,(\Delta u_t) \tag{6}$$

$$g_t\,(1 - \gamma_0 - \alpha\gamma_1 - \theta_t\gamma_2 - \theta_t\gamma_3) = \alpha\,(k_t) - \theta_t\,(\Delta u_t) \tag{7}$$

$$g_t = \frac{\alpha}{1 - \gamma_0 - \alpha\gamma_1 - \theta_t\gamma_2 - \theta_t\gamma_3}\,(k_t) - \frac{\theta_t}{1 - \gamma_0 - \alpha\gamma_1 - \theta_t\gamma_2 - \theta_t\gamma_3}\,(\Delta u_t) \tag{8}$$

$$g_t = \beta_{0,t} - \beta_{1,t}(\Delta u_t) + e_{1,t} \tag{9}$$

where $\beta_{0,t} = \alpha\,(k_t)\,/(1 - \gamma_0 - \alpha\gamma_1 - \theta_t\gamma_2 - \theta_t\gamma_3)$; $\beta_{1,t} = \theta_t/(1 - \gamma_0 - \alpha\gamma_1 - \theta_t\gamma_2 - \theta_t\gamma_3)$; and $e_{1,t}$ represents the stochastic disturbance term.

Equation (9) depicts the first difference version of Okun's law with time-varying parameters: $\beta_{1,t}$ represents the time-varying Okun coefficient on unemployment; and $\beta_{0,t}$ represents an estimate of a time-varying potential output growth rate. As different studies have pointed out (IMF , 2010; Klump et al. , 2008; Knotek , 2007; Lanzafame , 2010; León-Ledesma and Thirlwall , 2002; Mendieta-Muñoz , 2017; Schnabel , 2002; Thirlwall , 1969; Vogel , 2009), it is possible to assume that, when the u_t is constant —that is to say, when $\Delta u_t = 0$, then output is growing at its potential or "natural" rate (henceforth $g_{n,t}$), so that $g_{n,t} = \beta_{0,t}$.[3] This estimate represents the minimum level of output growth needed to reduce u_t given labour force and productivity growth. In other words, the rate of output growth consistent with a constant unemployment rate corresponds to the threshold growth rate, which, on a balanced growth path with no changes in unemployment, would equal the sum of the rates of growth of productivity and the labour force.[4]

[3] To the best of our knowledge, Thirlwall (1969) was the first to identify the rate of growth that keeps the unemployment rate constant with a measure of potential or "natural" output growth. This terminology stems from Roy Harrod's theoretical studies on the business cycle. Harrod (1939; 1960; 1970) defined the "natural" rate of growth as the "the maximum rate of growth allowed by the increase of population, accumulation of capital, technological improvement and the work leisure preference schedule, supposing that there is always full employment in some sense" (Harrod , 1939: 30). Therefore, it represents the "welfare optimum in which resources are fully employed and the best available technology used." (Harrod , 1960: 279), that is to say, it represents the "economic optimum growth rate" (Harrod , 1970: 737).

[4] Equation (9) also reverses the dependent and independent variables in the traditional Okun's law specification because, as Barreto and Howland (1993) emphasise, the research question determines the direction of regression, so that the best predictor of $g_{n,t}$ can be found by regressing g_t on Δu_t. Thirlwall (1969) also suggested reversing the dependent and independent variables in the traditional Okun's law specification in order to avoid estimation biases caused by labour hoarding.

Likewise, it is also possible to define a time-varying potential output per capita growth rate using the framework outlined above. Dividing both sides of equation (1) by total population (P):

$$Y/P = (AF(K,N))/P = (A/P)(F(kc,nh)) \tag{10}$$

where Y/P represents output per capita and A/P represents a measure of the technology level per capita.

Equation (10) can be expressed in growth rates as:

$$g_{p,t} = a_{p,t} + \alpha(k_t + c_t) + \theta(n_t + h_t) = a_{p,t} + \alpha(k_t + c_t) + \theta(l_t - \Delta u_t + h_t) \tag{11}$$

where $g_{p,t}$ and $a_{p,t}$ represent the rates of growth of output per capita and of multifactor productivity per capita, respectively.

In the same vein, if we incorporate the idea that the components of the rate of growth of potential output per capita can be affected by the actual output per capita growth rate ($g_{p,t}$) and a time-varying Okun coefficient on unemployment (θ_t), then equation (11) becomes:

$$g_{p,t} = \gamma_0'(g_{p,t}) + \alpha\left(k_t + \gamma_1'(g_{p,t})\right) + \theta_t\left(\gamma_2'(g_{p,t}) - \Delta u_t + \gamma_3'(g_{p,t})\right) \tag{12}$$

where $a_{p,t} = \gamma_0'(g_{p,t})$, $c_t = \gamma_1'(g_{p,t})$, $l_t = \gamma_2'(g_{p,t})$, and $h_t = \gamma_3'(g_{p,t})$, so that $\gamma_0' > 0$, $\gamma_1' > 0$, $\gamma_2' > 0$, $\gamma_3' > 0$ measure the degree of pro-cyclicality of $a_{p,t}$, c_t, l_t and h_t with respect to $g_{p,t}$.

Hence, equation (12) becomes:

$$g_{p,t} = \beta_{p,0,t} - \beta_{p,1,t}(\Delta u_t) + e_{p,1,t} \tag{13}$$

where $\beta_{p,0,t} = \alpha(k_t)/(1-\gamma_0' - \alpha\gamma_1' - \theta_t\gamma_2' - \theta_t\gamma_3')$; $\beta_{p,1,t} = \theta_t/(1-\gamma_0' - \alpha\gamma_1' - \theta_t\gamma_2' - \theta_t\gamma_3')$; and $e_{p,1,t}$ represents the error term.

In this sense, $\beta_{p,0,t}$ represents an estimate of a time-varying potential rate of growth of output per capita (henceforth $g_{p,n,t}$): it shows the per capita growth rate consistent with a constant unemployment rate —that is to say, the minimum level of output per capita growth needed to reduce the u_t.

To summarise, $g_{n,t} = \beta_{0,t}$ —from equation (9)— represents the time-varying potential output growth rate; whereas $g_{p,n,t} = \beta_{p,0,t}$ —from equation (13)— represents the time-varying potential output per capita growth rate.

2.2 The time-varying parameter models and a Heckman-type two step estimation procedure

Equations (9) and (13) can be estimated using a time-varying parameter model (TVPM), and we followed the standard procedure used to estimate the latter (Kim and Nelson , 1999). With respect to equation (9), the TVPM is composed of the observed variables Δu_t and g_t, and of the unobserved parameters $\beta_{0,t}$ and $\beta_{1,t}$:

$$g_t = \beta_{0,t} - \beta_{1,t}(\Delta u_t) + e_{1,t}, \quad e_{1,t} \sim i.i.d.\mathrm{N}(0, \sigma_e^2) \tag{14}$$

$$\beta_{i,t} = \beta_{i,t-1} + \varepsilon_{i,t}, \quad \varepsilon_{i,t} \sim i.i.d.\mathrm{N}(0, \sigma_{\varepsilon,i}^2), \quad i = 0, 1 \tag{15}$$

Likewise, the TVPM that represents equation (13) is composed of the observed variables Δu_t and $g_{p,t}$, and of the unobserved parameters $\beta_{p,0,t}$ and $\beta_{p,1,t}$:

$$g_{p,t} = \beta_{p,0,t} - \beta_{p,1,t}(\Delta u_t) + e_{p,1,t}, \quad e_{p,1,t} \sim i.i.d.\mathrm{N}(0, \sigma^2_{p,e}) \tag{16}$$

$$\beta_{p,i,t} = \beta_{p,i,t-1} + \varepsilon_{p,i,t}, \quad \varepsilon_{p,i,t} \sim i.i.d.\mathrm{N}(0, \sigma^2_{p,\varepsilon,i}), \quad i = 0,1 \tag{17}$$

However, one problem with the estimation of both TVPMs is that the regressor Δu_t may be correlated with $e_{1,t}$ and $e_{p,1,t}$ in equations (14) and (16), respectively, since both output and unemployment are endogenous variables to a complex system. The estimation of equations (14) and (15) and of equations (16) and (17) through the conventional Kalman filter via Maximum Likelihood (ML) cannot be performed because a successful application of the latter critically depends upon the assumption that the regressors are uncorrelated with the disturbance terms (Kim , 2006). In other words, the Kalman filter provides us with invalid inferences of the model if the regressors are endogenous.

In order to correct the possible endogeneity problem, we employ the Heckman-type two-step approach developed by Kim (2006), which allows us to obtain consistent estimates of both hyper-parameters and time-varying coefficients. Let us first illustrate this procedure using the TVPM shown in equations (14) and (15). We use Instrumental Variables (IVs) assuming that the relationship between the endogenous regressor Δu_t and the vector of IVs (z_t) is given by:

$$\Delta u_t = \delta(z_t) + \mu_t, \quad \mu_t \sim i.i.d.\mathrm{N}(0, \sigma^2_\mu) \tag{18}$$

where δ is a vector of constant parameters.[5]

It is possible to decompose Δu_t into its predicted component $(\mathrm{E}[\Delta u_t|\psi_{\mathbf{t-1}}])$ and its prediction error component (v_t):

$$\Delta u_t = \mathrm{E}[\Delta u_t|\psi_{\mathbf{t-1}}] + v_t \tag{19}$$

$$v_t = \sigma_v v^*_t, \quad v^*_t \sim i.i.d.\mathrm{N}(0,1) \tag{20}$$

where $\psi_{\mathbf{t-1}}$ denotes the available information in $t-1$; σ_v is the standard deviation of v_t; and v^*_t is the standardised prediction error of v_t.

If we denote the correlation between v_t and $e_{1,t}$ by the constant correlation coefficient ρ, the joint distribution of v^*_t and $e_{1,t}$ is the following:

$$\begin{bmatrix} v^*_t \\ e_{1,t} \end{bmatrix} \sim i.i.d.\mathrm{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho\sigma_e \\ \rho^{-1}\sigma_e & \sigma^2_e \end{bmatrix} \right) \tag{21}$$

Therefore, a Cholesky decomposition of the covariance matrix results leads us to decompose $e_{1,t}$ in equation (21) into:

$$e_{1,t} = \rho\sigma_e v^*_t + \omega^*_t, \quad \omega^*_t \sim i.i.d.\mathrm{N}(0, \sigma^2_\omega), \quad \sigma^2_\omega = (1-\rho^2)\sigma^2_e \tag{22}$$

[5] We also estimated equation (18) for all countries assuming that δ follows a random walk. However, it was not possible to find a global solution —that is, the solution went singular— when this specification was used. We also employed different starting values (using the OLS estimates as initial parameters); however, it was not possible to improve the results obtained using this specification.

Equation (22) shows the two components of $e_{1,t}$: the v_t^* component, which is correlated with Δu_t; and the ω_t^* component, which is not correlated with Δu_t. Substituting equation (22) into equation (14) results in:

$$g_t = \beta'_{0,t} - \beta'_{1,t}(\Delta u_t) + \rho \sigma_e v_t^* + \omega_t^* \tag{23}$$

where ω_t^* is not correlated with v_t^* or Δu_t; and both the new time-varying potential output growth rate ($g'_{n,t} = \beta'_{0,t}$) and the new time-varying Okun coefficient on unemployment ($\beta'_{1,t}$) can be generated assuming that the time-varying coefficients follow a random walk:

$$\beta'_{i,t} = \beta'_{i,t-1} + \varepsilon'_{i,t}, \quad \varepsilon'_{i,t} \sim i.i.d.\mathrm{N}(0, \sigma^{2'}_{\varepsilon,i}), \quad i = 0,1 \tag{24}$$

Thus, the standardised prediction errors v_t^* enter in equation (23) as bias correction terms in the spirit of Heckman (1976)'s two-step procedure for a sample selection model.

In the same vein, the TVPM that considers $g_{p,t}$ as dependent variable is estimated including v_t^* as bias correction terms in order to generate the new time-varying potential output per capita growth rate ($g'_{p,n,t} = \beta'_{p,0,t}$):

$$g_{p,t} = \beta'_{p,0,t} - \beta'_{p,1,t}(\Delta u_t) + \rho_p \sigma_{p,e} v_t^* + \omega_{p,t}^* \tag{25}$$

$$\beta'_{p,i,t} = \beta'_{p,i,t-1} + \varepsilon'_{p,i,t}, \quad \varepsilon'_{p,i,t} \sim i.i.d.\mathrm{N}(0, \sigma^{2'}_{p,\varepsilon,i}), \quad i = 0,1 \tag{26}$$

To summarise, the TVPMs with bias correction terms are estimated via ML in two steps:

1. Equation (18) is estimated through ML and the standardised one step-ahead forecast errors are obtained.

2. Equations (23) and (24) and equations (25) and (26) are estimated based on the prediction error decomposition and the Kalman filter.[6]

3 Results

Our initial purpose was to estimate the models for the G7 countries (Canada, France, Germany, Italy, Japan, the UK and the US). However, the u_t series available do not cover a long enough period in order to carry out the estimation for France and Italy. On the other hand, it was possible to estimate the models for Japan; however, it was not possible to find a global solution when these estimations were performed.[7] Given this, we restricted our attention to the estimation of the models for Canada, Germany, the UK and the US.

We used annual data for all countries, and the different estimation periods were selected according to the availability of data. The g_t and $g_{p,t}$ series were extracted from the Total Economy Database (TED) of the Groningen Development Centre. The u_t series for the different countries were extracted as follows: the OECD electronic database was employed for Canada and Germany; the Bank of England "300 Years of Data" dataset (for the period 1950-2008) and the

[6]The state-state representations of the TVPM without bias correction terms and of the TVPM with bias correction terms are presented in appendix A.

[7]This may be caused by non-linearities that are not captured by the TVPMs proposed. We leave the estimation for Japan for future research.

Office for National Statistics (for the period 2009-2014) were employed for the UK; and the Federal Reserve Economic Database of the St. Louis Fed (Fred) was employed for the US.[8] Regarding the IVs employed for Δu_t, we used different combinations of the lags of: Δu_t; the rate of growth of labour productivity measured as GDP per hour worked (r_t); the rate of growth of hours worked per person employed (hn_t); the change in the participation rate (Δpr_t); and the change in the working age population (between 15 and 64 years) as a percentage of total population (Δwp_t). The series for r_t, hn_t and total population for all countries were obtained from the TED; whereas the Δpr_t and working age population series were obtained from the OECD database.[9]

In all cases we proceeded as follows. We first ran the Kalman filter in order to obtain the respective innovation variances and the initial values of the parameters to be estimated in the different equitations. In the subsequent step, the Kalman filter was run again with the preceding estimates of the innovation variances, the initial values of the parameters and their respective variance-covariance matrices in order to obtain the evolutionary coefficients of the models.

Table 1 presents the estimates of the innovation variances for the state-space models without bias correction terms —equations (14) and (15) and equations (16) and (17); whereas Table 2 presents both the instrumental variable estimations for the different countries —equation (18)— and the estimates of the innovation variances for the models with bias correction terms —equations (23) and (24) and equations (25) and (26). Following Kim and Nelson (1999), we have also corroborated the appropriateness of the specified models checking for the lack of serial correlation and of heteroskedasticity in the standardized one-period-ahead-forecast errors of the different estimations. These results are presented in Table 3. Finally, Table 4 reports the Likelihood Ratio (LR) tests calculated assuming the respective models with constant parameters.

Firstly, from Table 3 it is possible to observe that the estimations of the different models do not present problems of serial correlation or heteroskedasticity (up to order 2) at the 5% level, which suggests no evidence of model misspecification.

Secondly, as mentioned before, different combinations of the lags (up to two) of Δu_t, r_t, hn_t, Δpr_t and Δwp_t were employed as instruments for Δu_t. The final combinations of instruments shown in Table 2 were selected according to two criteria based on the standard two-stage least square estimation: 1) the instruments employed needed to be valid (i.e., uncorrelated with the error term) according to Hansen's J-statistic; and 2) the instruments employed needed to be jointly significant according to the first-stage F-statistic.[10] The p-values associated with Hansen's J-statistic were the following: 0.72 for Canada; 0.19 for Germany; 0.10 for the UK; and 0.41 for the US. Hence, the joint null hypothesis of Hansen's J-test (that is, the instruments are valid and the excluded instruments are correctly excluded from the estimated equation) was not rejected in all countries. On the other hand, the p-values associated with the first-stage F-statistic were: 0.06 for Canada; 0.00 for Germany and the UK; and 0.02 for the US. Hence, the null hypothesis that the

[8]The u_t for the UK corresponds to the claimant count rate; whereas the u_t for the US corresponds to the civilian unemployment rate. For the US we also estimated the models using the OECD's u_t series and the g_t series obtained from the Fred. The results obtained were fairly similar.

[9]Δpr_t for the US refers to the change in the civilian labour force obtained from the Fred database.

[10]Hansen's J-statistic is a test for over-identifying restrictions that is consistent in the presence of heteroskedasticity and autocorrelation (Hayashi , 2000). On the other hand, for the case of a single endogenous regressor, the first-stage F-statistic corresponds to the Cragg–Donald F-statistic, which tests for weak identification (that is, it tests if instruments are only marginally relevant) (Stock and Yogo , 2005).

Table 1. Estimation of the hyper-parameters for the time-varying parameter models without bias correction terms

Hyper-parameters	Canada, 1956-2014	Germany, 1963-2014	United Kingdom, 1951-2014	United States, 1951-2014
Equations (14) and (15)[a]				
$\sigma_{\varepsilon,0}$	0.381***	0.428***	0.159	0.240
	(0.124)	(0.121)	(0.162)	(0.150)
$\sigma_{\varepsilon,1}$	0.045	0.521*	0.326**	0.074
	(0.164)	(0.276)	(0.152)	(0.100)
σ_e	0.919***	1.330***	1.321***	0.956***
	(0.108)	(0.148)	(0.108)	(0.088)
L^{b}	-106.203	-116.427	-132.400	-112.647
Equations (16) and (17)[a]				
$\sigma_{p,\varepsilon,0}$	0.429***	0.309**	0.193	0.227*
	(0.146)	(0.132)	(0.157)	(0.120)
$\sigma_{p,\varepsilon,1}$	0.016	0.434*	0.327**	0.076
	(0.349)	(0.244)	(0.152)	(0.086)
$\sigma_{p,e}$	0.936***	1.384***	1.301***	0.945***
	(0.112)	(0.147)	(0.117)	(0.092)
L^{b}	-108.120	-115.395	-132.237	-111.717

Notes: [a]Standard errors are shown in parenthesis; [b]Log likelihood.
*, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Table 2. Estimation of the hyper-parameters for the instrumental variables and for the time-varying parameter models with bias correction terms

Hyper-parameters	Canada, 1956-2014	Germany, 1963-2014	United Kingdom, 1951-2014	United States, 1951-2014
Instrumental variable estimation: equation (18)[a]				
Instruments employed for Δu_t	$\Delta u_{t-1}, hn_{t-1}$	$\Delta u_{t-1}, \Delta u_{t-2}, \Delta pr_{t-2}, \Delta wp_{t-2}$	$\Delta u_{t-1}, hn_{t-1}$	hn_{t-1}, hn_{t-2}
σ_μ	0.927***	0.636***	0.664***	1.000***
	(0.066)	(0.067)	(0.039)	(0.065)
L^{b}	-102.840	-89.865	-89.877	-114.405
Equations (23) and (24)[a]				
$\sigma'_{\varepsilon,0}$	0.442***	0.478***	0.133	0.248
	(0.133)	(0.130)	(0.175)	(0.157)
$\sigma'_{\varepsilon,1}$	0.064	0.586**	0.316*	0.079
	(0.140)	(0.291)	(0.172)	(0.098)
$\sigma_{\omega*}$	0.830***	1.254***	1.304***	0.955***
	(0.106)	(0.172)	(0.122)	(0.096)
ρ	-0.332	-0.229	-0.588	0.259
	(0.250)	(0.482)	(0.389)	(0.180)
L^{b}	-102.409	-111.907	-129.237	-109.849
Equations (25) and (26)[a]				
$\sigma'_{p,\varepsilon,0}$	0.561***	0.325**	0.165	0.227*
	(0.163)	(0.142)	(0.163)	(0.127)
$\sigma'_{p,\varepsilon,1}$	0.036	0.487**	0.314*	0.084
	(0.187)	(0.247)	(0.168)	(0.082)
$\sigma_{p,\omega*}$	0.793***	1.335***	1.286***	0.944***
	(0.124)	(0.162)	(0.133)	(0.103)
ρ_p	-0.471*	-0.282	-0.596	0.295
	(0.260)	(0.468)	(0.385)	(0.180)
L^{b}	-104.012	-110.949	-129.075	-108.789

Notes: [a]Standard errors are shown in parenthesis; [b]Log likelihood.
*, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Table 3. Correct specification tests on the one-period-ahead forecast errors obtained from the time-varying parameter models

	Autocorrelation[a]		Heteroskedasticity[b]	
	Order 1	Order 2	Order 1	Order 2
Models without bias correction terms: equations (14) and (15)				
Canada	0.86	0.98	0.58	0.51
Germany	0.67	0.40	0.21	0.42
United Kingdom	0.07	0.07	0.89	0.99
United States	0.63	0.89	0.52	0.34
Models without bias correction terms: equations (16) and (17)				
Canada	0.99	0.94	0.77	0.25
Germany	0.86	0.33	0.34	0.58
United Kingdom	0.08	0.07	0.51	0.78
United States	0.82	0.97	0.32	0.20
Instrumental variable estimation: equation (18)				
Canada	0.08	0.22	0.39	0.65
Germany	0.44	0.68	0.07	0.22
United Kingdom	0.12	0.10	0.95	0.86
United States	0.54	0.29	0.63	0.88
Models with bias correction terms: equations (23) and (24)				
Canada	0.65	0.90	0.52	0.68
Germany	0.78	0.51	0.19	0.36
United Kingdom	0.15	0.15	0.70	0.76
United States	0.57	0.85	0.53	0.40
Models with bias correction terms: equations (25) and (26)				
Canada	0.62	0.88	0.41	0.50
Germany	0.98	0.39	0.34	0.58
United Kingdom	0.15	0.16	0.89	0.55
United States	0.81	0.95	0.26	0.21

Notes: [b]*P*-values of the Ljung-Box statistics (*Ho*: no serial correlation); [c]*P*-values of the ARCH tests (*Ho*: no autoregressive conditional heteroskedasticity effects).

Table 4. Log likelihoods of the Time-Varying Parameter Models (TVPM) and of the Constant Parameter Models (CPM), and Likelihood Ratio (LR) tests

	Canada 1956-2014	Germany 1963-2014	United Kingdom 1951-2014	United States 1951-2014
Models without bias correction terms: equations (14) and (15)				
TVPM	-106.203	-116.427	-132.400	-112.647
CPM	-128.509	-127.734	-135.548	-123.318
LR statistic	44.61	22.61	6.29	21.34
p-value	0***	0***	0.04**	0***
Models without bias correction terms: equations (16) and (17)				
TVPM	-108.120	-115.395	-132.237	-111.717
CPM	-120.959	-123.148	-135.902	-118.263
LR statistic	25.68	15.51	7.33	13.09
p-value	0***	0***	0.03**	0***
Models with bias correction terms: equations (23) and (24)				
TVPM	-102.409	-111.907	-129.237	-109.849
CP	-124.997	-122.190	-131.611	-120.542
LR statistic	45.17	20.57	4.75	21.39
p-value	0***	0***	0.09*	0***
Models with bias correction terms: equations (25) and (26)				
TVPM	-104.012	-110.949	-129.075	-108.789
CPM	-118.806	-118.170	-131.779	-115.732
LR statistic	29.59	14.44	5.41	13.89
p-value	0***	0***	0.07*	0***

*, **, and *** denote rejection of the null hypothesis of the LR test (*Ho*: the smaller CPM is the "true" model) at the 10%, 5%, and 1% levels, respectively.

instruments employed are jointly non-significant was rejected in all countries (at the 10% level).

Thirdly, regarding the endogeneity problem of the regressor Δu_t, from Table 2 it is also possible to observe that the estimated coefficient of the correction term bias ρ is statistically non-significant in all estimations; whereas the coefficient ρ_p is statistically significant only in Canada. Hence, with the exception of Canada when $g_{p,t}$ was used as dependent variable, it is possible to ignore the endogeneity problem. In other words, in order to retrieve the potential output growth rate estimates from the TVPMs it is necessary to consider equations (14) and (15) for all countries; whereas to retrieve the potential per capita output growth rate estimates from the TVPMs it is necessary to consider equations (25) and (26) for Canada, and equations (16) and (17) for all other countries.

Fourthly, as regards the estimates of the standard errors associated with $g_{n,t} = \beta_{0,t}$ (that is, $\sigma_{\varepsilon,0}$), Table 1 shows that these are statistically significant in Canada and Germany, thus suggesting evidence of a temporary variation in the potential rate of growth. The $\sigma_{\varepsilon,0}$s were found to be statistically non-significant in the UK and in the US. However, as shown in Table 4, the Likelihood Ratio (LR) tests calculated assuming the respective models with constant parameters rejects the null hypothesis of constant parameters at the 5% level in all countries. In this sense, the LR tests suggest that it is also preferable to consider the models with time-varying parameters for the UK and the US.

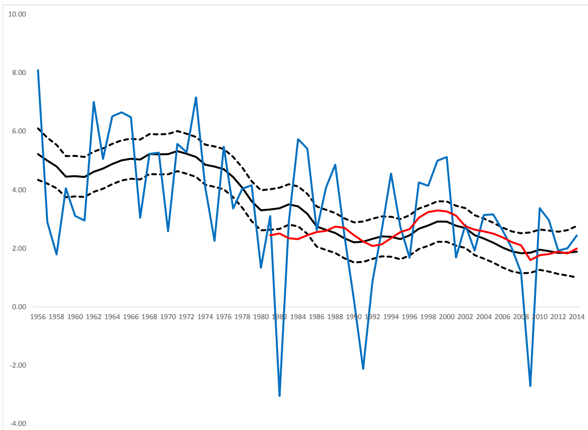
Finally, with respect to the estimates of the standard errors associated with $g_{p,n,t} = \beta_{p,0,t}$ ($g'_{p,n,t} = \beta'_{p,0,t}$ for Canada), Table 1 (Table 2 for Canada) shows that the $\sigma_{p,\varepsilon,0}$s ($\sigma'_{p,\varepsilon,0}$ for Canada) are statistically significant in all countries, the only exception being the UK. However, once again the LR test calculated assuming the respective model with constant parameters in the UK —shown in Table 4— rejects the null hypothesis of constant parameters at the 5% level, so that it is also preferable to consider the TVPM for the UK. From Table 4 it is also possible to observe that the LR tests for the rest of the countries reject the null hypothesis of constant parameters at the 5% level.

The time-varying potential output growth rates are presented in Figures 1a to 1d; whereas the time-varying potential output per capita growth rates are presented in Figures 2a to 2d.[11] All figures show the smoothed estimates of the $g_{n,t}$s and the $g_{p,n,t}$s ($g'_{p,n,t}$ for Canada), together with their respective 90% confidence intervals.[12] Figures 1a to 1d also plot the rates of growth of potential output estimated by the IMF —available only for the period 1981-2014, and by the Congressional Budget Office (CBO) —available only for the US. It is worth noting that, with few exceptions,

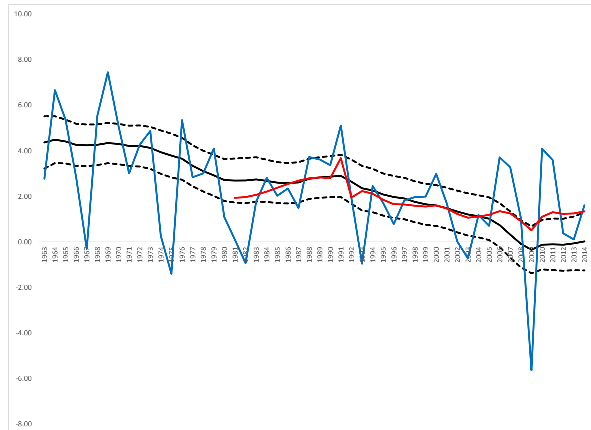
[11] Appendix B presents the time-varying Okun coefficients on unemployment. These empirical results need to be interpreted in the light of a mix of components such as the demographic structure of the country, its labour market flexibility, its labour market policies, and its policy implementation timing. This exceeds the purpose of the current paper. Nevertheless, it is possible to say that the results obtained corroborate previous findings: using a penalized regression spline estimator for the period 1981-2011, Mendieta-Muñoz (2017) reports an increase in the Okun coefficient on unemployment in Canada (from around -2.1% to around -2.2%), Germany (from around -0.8% to around -1.3%) and the UK (from around -0.8% to around -2.5%), and a reduction in the Okun coefficient in the US (from around -2.0% to around -1.4%); whereas Daly et al. (2014) use rolling regressions (40-quarter rolling window) during the period 1949Q1–2014Q1 for the US economy, finding a reduction in the Okun coefficient on unemployment from around -2.1% to around -1.9%.

[12] Depending upon the information set used, it is possible to find the basic filter and smoothing filter. The former refers to an estimate of the time-varying coefficients based on information available up to time t; whereas the latter refers to an estimate of the time-varying coefficients based on all the available information in the sample through time T. The smoothed values provide a more accurate inference about the time-varying parameters (see Kim and Nelson (1999) and Kim (2006) for a description).

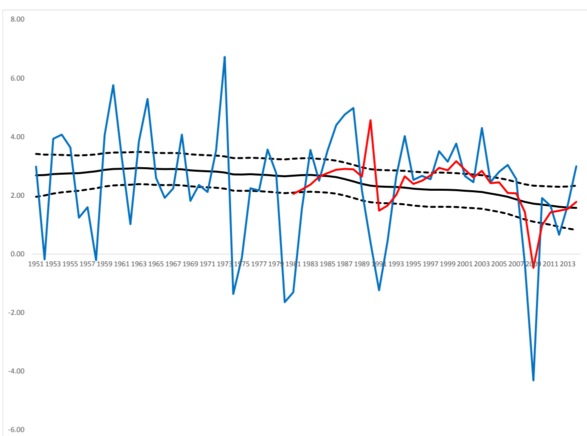
both the IMF's and the CBO's estimates of $g_{n,t}$ lie within our estimated 90% confidence intervals during the periods of study. On the other hand, to the best of our knowledge, there are no potential output per capita growth estimates provided by international organisations, so that it is not possible to compare our results with any other measures.



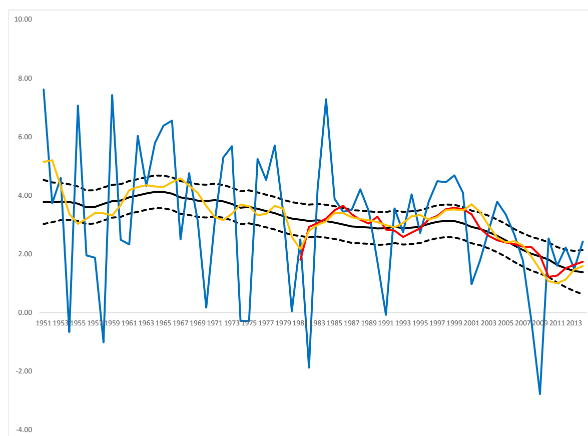
(a) Canada, 1956-2014



(b) Germany, 1963-2014

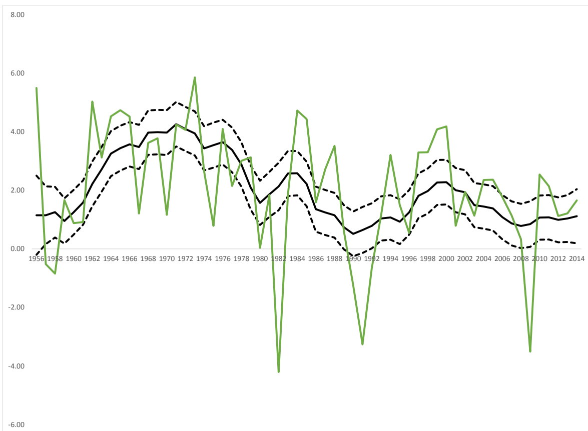


(c) UK, 1951-2014

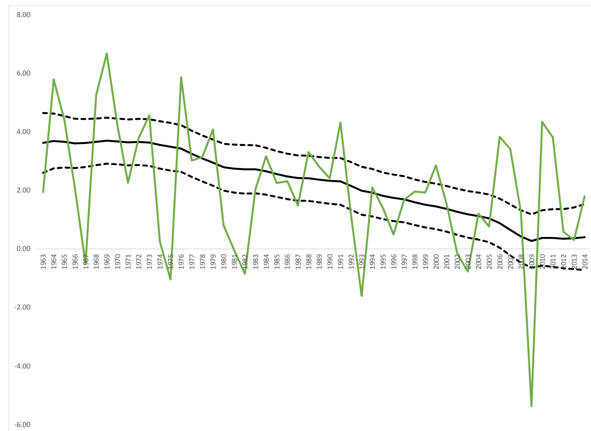


(d) US, 1951-2014

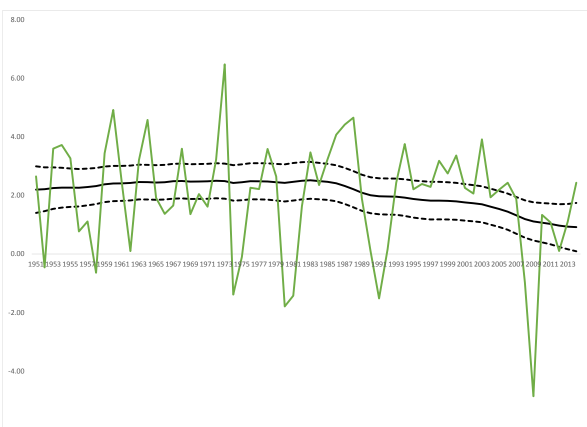
Figure 1. Time-varying potential output growth rates (black straight lines) with 90% confidence intervals (black dotted lines); actual growth rates (blue straight lines); IMF's potential output growth rates (red straight lines); and CBO's potential output growth rate for the US (yellow straight line)



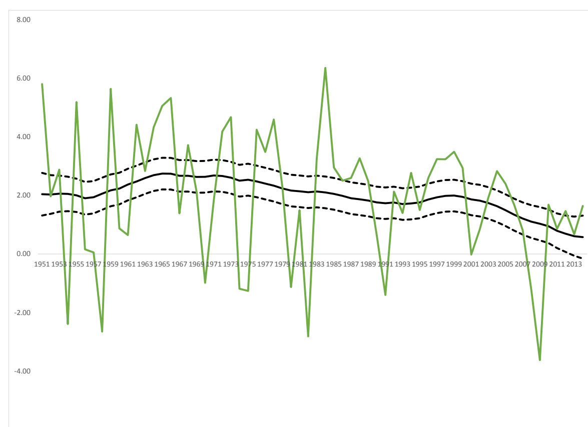
(a) Canada, 1956-2014



(b) Germany, 1963-2014



(c) UK, 1951-2014



(d) US, 1951-2014

Figure 2. Time-varying potential output per capita growth rates (black straight lines) with 90% confidence intervals (black dotted lines) and actual per capita growth rate (green straight lines)

Finally, in order to provide a more comprehensive analysis of the behaviour of the series presented in Figures 1 and 2, table 5 below calculates the percentage point (pp) changes in the estimated potential output growth rates, together with the pp changes in the population growth rates for each country, both for the complete periods of study and for the periods up until 2007 (that is, before the GR):

Table 5. Percentage point changes in the estimated time-varying potential output growth rates and population growth rates

Percentage point changes in the growth rates of:	*Canada*	*Germany*	*United Kingdom*	*United States*
	1956-2014	*1963-2014*	*1951-2014*	*1951-2014*
Potential output	-3.33	-4.34	-1.11	-2.39
Potential output per capita	-0.03	-3.22	-1.28	-1.47
Population	-1.69	-1.00	0.22	-0.94
	1956-2007	*1963-2007*	*1951-2007*	*1951-2007*
Potential output	-3.32	-4.05	-0.82	-1.64
Potential output per capita	-0.28	-2.97	-0.88	-0.83
Population	-1.60	-0.95	0.34	-0.76

Upon inspection of Figures 1 and 2 and Table 5, it is possible to summarise the main findings as follows:

1. The $g_{n,t}$s have fallen in the four countries of study during the respective periods: Germany is the country that has experienced the largest reduction in this measure of potential output growth (-4.3 pp), followed by Canada (-3.3 pp), the US (-2.4 pp), and the UK (-1.1 pp).

2. Most of the slowdown in the $g_{n,t}$s occurred before the GR: -4.1 pp in Germany, -3.3 pp in Canada, -1.6 pp in the US, and -0.8 pp in the UK.

3. The $g'_{p,n,t}$ in Canada has remained virtually unchanged during the period 1956-2014.

4. The $g_{p,n,t}$s have fallen in Germany (-3.2 pp), the US (-1.5 pp) and the UK (-1.3 pp).

5. Most of the slowdown in the $g_{p,n,t}$s also occurred before the GR in Germany (-2.9 pp), the US (-0.8 pp) and the UK (-0.9 pp).

6. The fall in the $g_{p,n,t}$s is less than the fall in the $g_{n,t}$s in Germany and the US; whereas the fall in $g_{p,n,t}$ is larger than the fall in $g_{n,t}$ in the UK.

Therefore, it is possible to derive the following conclusions from our analysis:

1. Most of the slowdown in potential output growth and in potential output per capita growth occurred before the GR.

2. Canada presents an important difference between the estimated potential output growth rate and the estimated potential output per capita growth rate since the former has fallen during

the period 1956-2014; whereas the latter has remained virtually unchanged. This means that the reduction in the productive capacity in Canada seems to be mainly associated with the fall in the rate of growth of population, which has fallen by approximately -1.7 pp, the largest fall in population growth out of the four countries of study.

3. The fall in the rate of growth of population is an important factor in order to explain the reduction of the productive capacity in Germany and the US, but not in the UK. Indeed, population growth increased by approximately 0.2 pp in the UK (1951-2014); whereas it decreased by -1.0 pp in Germany (1963-2014) and by -0.9 pp in the US (1951-2014).

3.1 A comparison with the Hodrick-Prescott filter

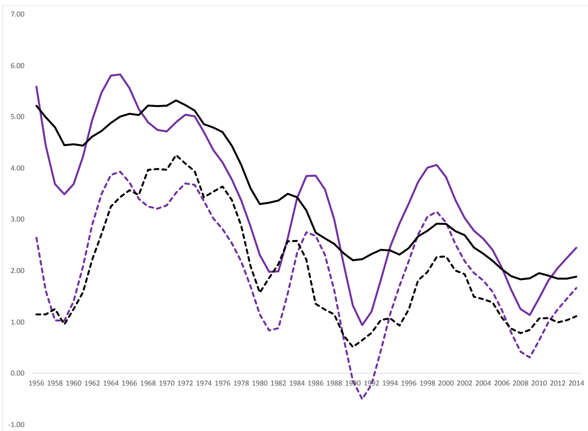
We finally compare the results obtained from the TVPMs with those obtained from the HP filter. The latter was used to obtain the trend components of the g_t and $g_{p,t}$ series following the suggestion proposed by Ravn and Uhlig (2002) for annual data, so that the smoothing parameter was selected to be 6.25. These results are presented in Figures 3a to 3d, which also plot the potential growth rates obtained from the TVPMs. Firstly, it is possible to observe that the trend component of the g_t and $g_{p,t}$ series obtained using the HP filter exhibit stronger fluctuations over time in the UK and the US. Secondly, according to the HP filter, the fall in the trend component of the g_t series in the US has been approximately -3.8 pp (1951-2014) and -4.9 pp (1951-2007); -3.3 pp (both in 1963-2014 and in 1963-2007) in Germany; -3.1 pp (1956-2014) and -3.9 pp (1956-2007) in Canada; and -0.9 pp and -0.7 pp (1951-2014) in the UK. Finally, the fall in the trend component of the $g_{p,t}$ series according to the HP filter has been: -2.8 pp (1951-2014) and -4.0 (1951-2007) in the US; -1.9 pp (1964-2014) and -1.8 pp (1963-2007) in Germany; -1.3 pp (1951-2014) and -1.0 pp (1951-2007) in the UK; and -0.9 pp (1956-2014) and -1.9 pp (1956-2007) in Canada.

The drawbacks to the HP filtering approach have been known for some time (see Hamilton (2016) for a recent paper). However, the HP filter also shows: 1) a reduction in the trend components of the g_t and $g_{p,t}$ series; and 2) that the fall in the trend component of the $g_{p,t}$ series has been less than the fall in the the trend component of the g_t series in Germany and the US; whereas the latter has fallen less than the former in the UK.

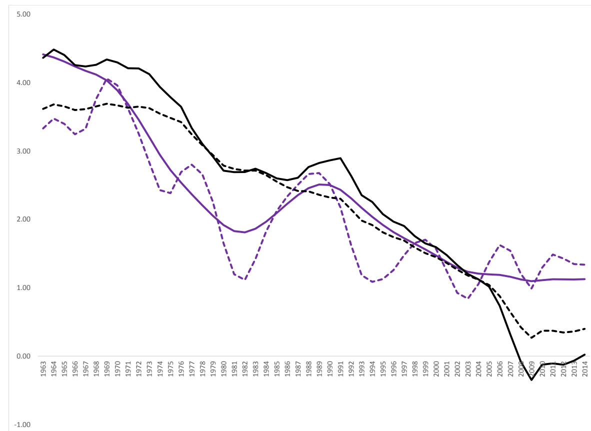
Regarding Canada, the HP filter shows a reduction in the trend component of the $g_{p,t}$ series. Indeed, it is necessary to emphasise that the only estimation that presented endogeneity problems according to the TVPMs employed in the previous section was precisely Canada when the time-varying potential output per capita growth rate was estimated (that is to say, when the $g_{p,t}$ was used as dependent variable). Thereby, the endogeneity problem seems to be an extremely important element that is not captured adequately by the HP filter.

4 Concluding remarks

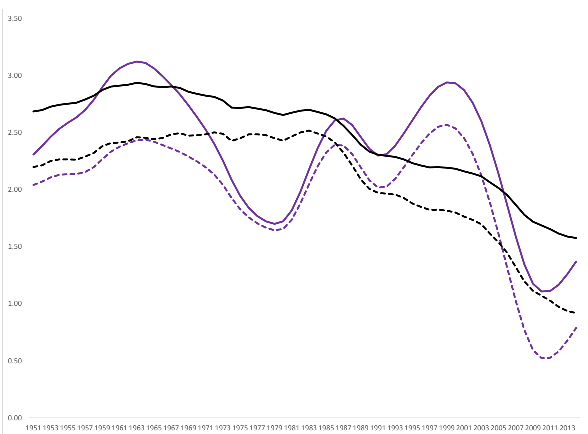
This paper has compared the evolution of a measure of potential output growth —the rate of growth of output that is consistent with a stable unemployment rate— with the evolution of a measure of potential output per capita growth —the rate of growth of output per capita that is consistent with a stable unemployment rate— in Canada (1956-2014), Germany (1963-2014), the United Kingdom (1951-2014) and the United States (1951-2014). We estimated both potential



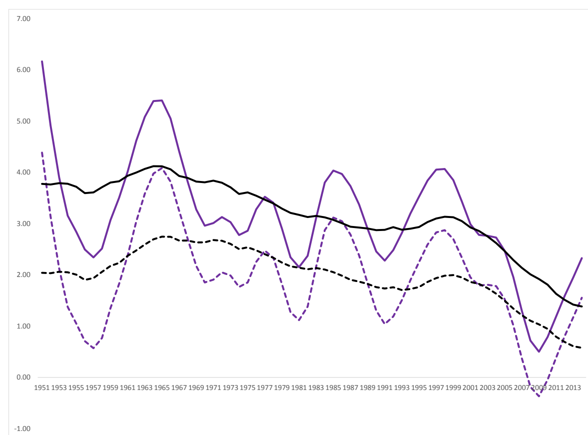
(a) Canada, 1956-2014



(b) Germany, 1963-2014



(c) UK, 1951-2014



(d) US, 1951-2014

Figure 3. Time-varying potential output growth rates (black straight lines); time-varying potential output per capita growth rates (black dotted lines); and trend components of the actual growth rate (purple straight lines) and actual per capita growth rate (purple dotted lines) obtained from the Hodrick-Prescott filter

output growth rates using time-varying parameter models and a Heckman-type two-step estimation procedure that deals with the issue of endogenous regressors, finding differences in the evolution of both potential growth rates. Our evidence supports the view that the potential output growth rate has fallen in the four countries of study, and that most of the slowdown occurred prior to the Great Recession. On the other hand, the potential output per capita growth rate has decreased only in Germany, the United States and the United Kingdom; whereas it has remained remained relatively constant in Canada. Finally, the decline in the potential output per capita growth rate has been less than the respective decline in the potential output growth rate in Germany and the United States; whereas it has been larger in the United Kingdom.

The empirical results indicate the existence of two important characteristics regarding the behaviour of the productive capacity in these four developed economies. Firstly, the decline in potential output growth in Canada seems to be mainly associated with the fall in population growth —approximately -1.7 percentage points during 1956-2014, which represents the largest fall in population growth out of the four countries of study. Secondly, the deceleration of population growth seems to be an important factor in order to explain the decline in potential output growth in Germany and in the United States, but not in the United Kingdom —where population growth has slightly increased.

The results found raise questions about the underlying properties of output and about how different models specify the behaviour of potential output around recessions and expansions. Future theoretical and empirical research should try to offer a more detailed analysis of the relevant short-run and/or medium-run fluctuations that may affect the individual components of potential output growth, such as rising income inequality and the growth of the financial sector. In this sense, more studies are needed in order to provide a better understanding of the deep causes of the secular decline in growth, given the individual characteristics exhibited by each country.

References

Antolin-Diaz, J., Drechsel, T. and Petrella, I. (2016). Tracking the slowdown in long-run GDP growth. *Bank of England Staff Working Paper*, No. 587.

Arrow, K. (1962). The economic implications of learning by doing. *The Review of Economic Studies*, 29, 155-173.

Barreto, Humberto and Howland, F. (1993). There are two Okun's law relationship between output and unemployment. *Wabash College Working Paper*.

Basu, S. and Fernald, J. (2009). What do we know (and not know) about potential output? *Federal Reserve Bank of St. Louis Review*, July-August, 187-214.

Benati, L. (2007). Drift and breaks in labor productivity. *Journal of Economic Dynamics and Control*, 31, 2847-2877.

Blanchard, O., Cerutti, E. and Summers, L. (2015). Inflation and activity —Two explorations and the monetary policy implications. *NBER Working Paper*, No. 21726.

Byrne, D., Fernald, J. and Reinsdorf, M. (2016). Does the United States have a productivity slowdown or a measurement problem? *Federal Reserve Bank of San Francisco Working Paper*, No. 2016-03.

Cerra, V. and Saxena, S. (2000). Alternative methods of estimating potential output and the output gap: An application to Sweden. *IMF working paper*. WP/00/59.

Cerra, V. and Saxena, S. (2008). Growth dynamics: The myth of economic recovery. *American Economic Review*, 98, 439-457.

Cette, G., Fernald, J. and Mojon, B. (2016). The pre-Great Recession slowdown in productivity. *Federal Reserve Bank of San Francisco Working Paper*, No. 2016-08.

Daly, M., Fernald, J., Jordà. Ò. and Nechio, F. (2014). Output and unemployment dynamics. *Federal Reserve of San Francisco Working Paper*, No. 2014-32.

DeLong, B. and Summers, L. (2012). Fiscal policy in a depressed economy. *Brooking Papers on Economic Activity*, Spring, 233-274.

European Central Bank (ECB). (2011). Trends in potential output. *ECB Monthly Bulletin*. January, 73-85.

Eggertsson, G. and Mehrotra, N. (2014). A model of secular stagnation. *NBER Working Paper*, No. 20574.

Fernald, J. (2007). Trend breaks, long-run restrictions, and contractionary technology improvements. *Journal of Monetary Economics*, 54, 2467-2485.

Fernald, J. (2014). Productivity and potential output before, during, and after the Great Recession. *NBER Macroeconomics Annual*, 29, 1-51.

Fernald, J. and Wang, B. (2015). The recent rise and fall of rapid productivity growth. *FRBSF Economic Letter*, No. 2015-04.

Galí, J., Smets, F. and Wouters, R. (2012). Slow recoveries: A structural interpretation. *Journal of Money, Credit and Banking*, 44, 9-30.

Gordon, R. (2012). Is US economic growth over? Faltering innovation confronts the six headwinds. *NBER Working Paper*, No. 18315.

Gordon, R. (2013). U.S. Productivity growth: The slowdown has returned after a temporary revival. *International Productivity Monitor*, 25, 13-19.

Gordon, R. (2014a). The demise of U.S. economic growth: Restatement, rebuttal and reflections. *NBER Working Papers*, No. 19895.

Gordon, R. (2014b). A new method of estimating potential real GDP growth: Implications for the labor market and the debt/GDP ratio. *NBER Working Papers* No. 20423.

Gordon, R. (2015). Secular stagnation on the supply side: U.S. productivity growth in the long run. *Digiworld Economic Journal*, 100, 19-44.

Hamilton, J. (2016). Why you should never use the Hodrick-Prescott filter. *Working Paper*. Available at http://econweb.ucsd.edu/~jhamilto/hp.pdf.

Harrod, R. (1939). An essay in dynamic theory. *Economic Journal*, 49, 14-33.

Harrod, R. (1960). Second essay in dynamic theory. *Economic Journal*, 70, 277-293.

Harrod, R. (1970). Harrod after twenty-one years. A comment. *Economic Journal*, 81, 737-741.

Hayashi, F. (2000). *Econometrics*. Princeton: Princeton University Press.

Heckman, J. (1976). The common structure of statistical models of truncation, sample selection, and limited dependent variables and a simple estimator for such models. *Annals of Economic and Social Measurement*, 5, 475-492.

Huang, H. and Lin, S. (2008). Smooth time-varying Okun's coefficient. *Economic Modelling*, 25, 363–375.

International Monetary Fund. (2010). *World Economic Outlook. Rebalancing Growth*, 69-107.

Kim, C. (2006). Time-varying parameter models with endogenous regressors. *Economics Letters*, 91: 21-26.

Kim, C. and Nelson, C. (1999). *State-space Models with Regime Switching. Classical and Gibbs-sampling Approaches with Applications*. Cambridge: The MIT Press.

Klump, R., McAdam, P. and Willman, A. (2008). Unwrapping some euro area growth puzzles: Factor substitution, productivity and unemployment. *Journal of Macroeconomics*, 30, 645-666.

Knotek, E. (2007). How useful is Okun's law? *Federal Reserve Bank of Kansas City Economic Review*, 4, 73-103.

Lanzafame, M. (2010). The endogeneity of the natural rate of growth in the regions of Italy. *International Review of Applied Economics*, 24, 533-552.

León-Ledesma, M. and Thirlwall, A. (2002). The endogeneity of the natural rate of growth. *Cambridge Journal of Economics*, 26, 441-459.

Martin, R., Munyan, T. and Wilson, B. (2015). Potential output and recessions: Are we fooling ourselves? *International Finance Discussion Papers*, No. 145.

Mendieta-Muñoz, I. (2017). On the interaction between economic growth and business cycles. *Macroeconomic Dynamics*, forthcoming.

Ravn, M. and Uhlig, H. (2002). On adjusting the Hodrick–Prescott filter for the frequency of observations. *The Review of Economics and Statistics*, 84, 371–375.

Schnabel, G. (2002). Output trends and Okun's law. *Bank for International Settlements (BIS), Working Paper*, No. 111.

Stock, James H. and Motohiro, Y. (2005). Testing for weak instruments in linear IV regression. In Andrews, D. and Stock, J. (eds.). *Identification and Inference for Econometric Models: Essays in Honor of Thomas Rothenberg*, 80-108. Cambridge: Cambridge University Press.

Stock, J. and Watson, M. (2012). Disentangling the channels of the 2007-09 recession. *Brookings Papers on Economic Activity*, Spring, 81-135.

Thirlwall, A. (1969). Okun's law and the natural rate of growth. *Southern Economic Journal*, 36, 87-89.

Vogel, L. (2009). The endogeneity of the natural rate of growth – an empirical study for Latin-American countries. *International Review of Applied Economics*, 23, 41-53.

A State-state representation of the time-varying parameter models

We only show the model that uses g_t as dependent variable since the representation using $g_{p,t}$ is similar. The state-space formulation of the time-varying parameter model without bias correction terms —equations (14) and (15)— is the following:

$$\mathbf{Y_t} = \mathbf{X_t}\mathbf{B_t} + \mathbf{e_t}, \quad \mathbf{e_t} \sim i.i.d.\mathrm{N}(0, \sigma_e^2) \tag{A.1}$$

$$\mathbf{B_t} = \mathbf{B_{t-1}} + \varepsilon_\mathbf{t}, \quad \varepsilon_\mathbf{t} \sim i.i.d.\mathrm{N}(0, \Sigma_\varepsilon) \tag{A.2}$$

where $\mathbf{Y_t} = [g_t]$; $\mathbf{X_t} = [1, \Delta u_t]$; $\mathbf{B_t} = \begin{bmatrix} \beta_{0,t} \\ \beta_{1,t} \end{bmatrix}$; $\mathbf{e_t} = [e_{2,t}]$; $\mathbf{B_{t-1}} = \begin{bmatrix} \beta_{0,t-1} \\ \beta_{1,t-1} \end{bmatrix}$; and $\varepsilon_\mathbf{t} = \begin{bmatrix} \varepsilon_0 \\ \varepsilon_1 \end{bmatrix}$.

On the other hand, the state-state representation of the model with bias correction terms —equations (23) and (24)— is the following:

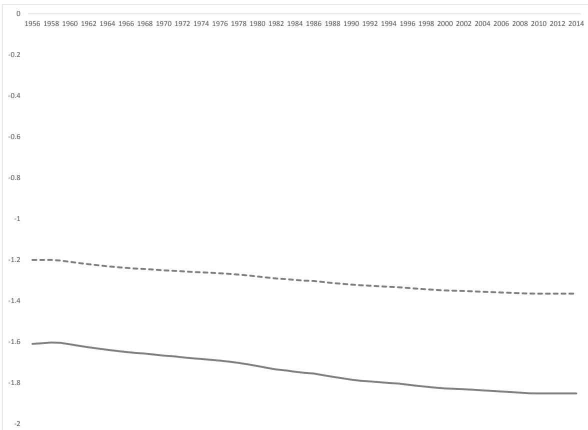
$$\mathbf{Y_t} = \mathbf{X_t}\mathbf{B'_t} + \rho\sigma_e v_t^* + \omega_t^*, \quad \omega_t^* \sim i.i.d.\mathrm{N}(0, (1-\rho^2)\sigma_e^2) \tag{A.3}$$

$$\mathbf{B'_t} = \mathbf{B'_{t-1}} + \varepsilon'_\mathbf{t}, \quad \varepsilon'_\mathbf{t} \sim i.i.d.\mathrm{N}(0, \Sigma'_\varepsilon) \tag{A.4}$$

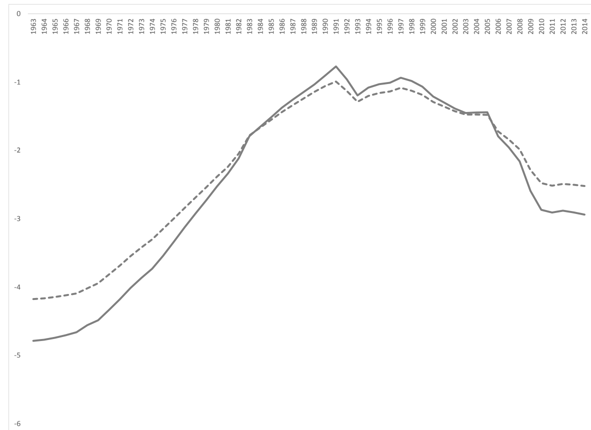
where, in addition to the previously defined variables, we now have that $\mathbf{B'_t} = \begin{bmatrix} \beta'_{0,t} \\ \beta'_{1,t} \end{bmatrix}$; $\mathbf{B_{t-1}} = \begin{bmatrix} \beta'_{0,t-1} \\ \beta'_{1,t-1} \end{bmatrix}$; $\varepsilon'_\mathbf{t} = \begin{bmatrix} \varepsilon'_0 \\ \varepsilon'_1 \end{bmatrix}$.

Thus, equations (A.1) and (A.3) represent the measurement equations of the models; whereas equations (A.2) and (A.4) represent the transition equations.

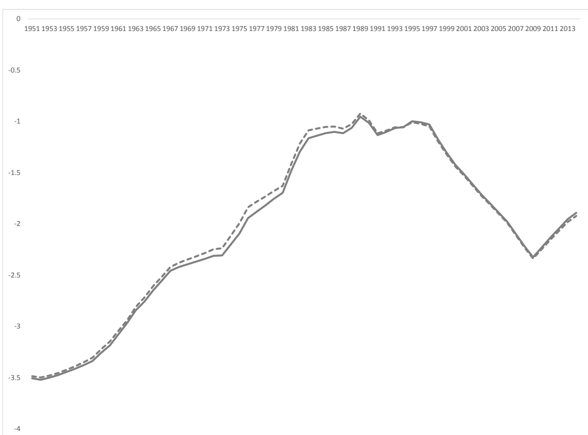
B Time-varying Okun coefficients on unemployment



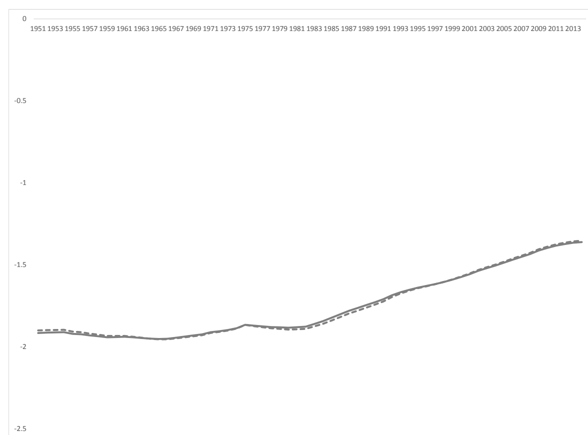
(a) Canada, 1956-2014



(b) Germany, 1963-2014



(c) UK, 1951-2014



(d) US, 1951-2014

Figure B.1. Time-varying Okun coefficients on unemployment: using the actual growth rate (grey straight line) and the actual per capita growth rate (grey dotted line) as dependent variables