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A Simple Dynamic-Control Macro Model to Examine the Behavior of International Reserves for Selected Economies

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Abstract

The purpose of the paper is to show the construction of a simple dynamic-control macro model, using an economy-wide preference (utility) function as the objective function with two variables, national income and international reserves. National income is the control variable and reserves is the state variable. The first-order equilibrium condition at each instant of time, t, shows that the two variables optimally must grow at different rates for it to be satisfied. The model is applied to the empirical data on income and reserves by examining the behavior of the ratio of income to reserves for a selected number of mature and developing economies over the time period 1970 to 2011. For both types of economies, the model shows that there is a trade-off between income and reserves based on the utility function. However, for mature economies the trade-off is such that the ratio of income to reserves is trending upwards while for the developing economies the ratio is trending downwards. In either case, the model fits the data. The policy implications are briefly discussed within the context of the existing literature on international reserves.

Keywords: Macro dynamic control, Income, Reserves, Empirical ratios
JEL Classification: C61; F3; F4

1. Introduction

The purpose of this paper is to see if the macro economy, subject to all kinds of economic and political forces, behaves, as if, it is a dynamic control theory model. The paper is a special application of dynamic control theory along the lines found in Intriligator's (1971) dynamic macro model. The difference is that the present model focuses on international reserves and national income. As with any dynamic control theory model, of significance is the behavior of the model's shadow price. This price, as is well known, reflects the trade-off between current decisions and future outcomes. The paper does not estimate empirically this price, but it uses its theoretical behavior to hypothesize about the empirical behavior of the model's decision variable and state variable.

In the case of the paper here, the trade-off is between national income and international reserves. As will be shown, both variables affect national utility, but more income means due to the propensity to import less international reserves (all other things given). Forgoing income means more international reserves. If the economy is maximizing its utility (a common goal in most macro models), then the behavior of income relative to reserves becomes an interesting empirical question and also a question of relevance to the dynamic model behind the behavior of the two variables. The behavior of this ratio (income to reserves) also has important economic policy issues for the economy involved. For example, why does an economy self-insure with respect to the rest of the world? This is explored later.

The literature directly relevant to the paper is sparse but more generally can be divided into two parts: one methodological (the control theory) and the other part policy (why have reserves?). The first part is quite large, very mechanistic and developed during the 1950's, 1960's and 1970's. The earlier reference to the Intriligator's model (1971) and much of the macro control theory literature on capital accumulation cited in his bibliography to chapter 16 on optimal economic growth gives examples of dynamic optimal macro control theory. This literature has general background relevance to the present paper, but it is not specific to the particular model developed here, so it is not explicitly reviewed.

The second part of the literature is also very large and very current. Much of it is from the NBER, the OECD, and the IMF. It is concerned with reserves as self-insurance against sudden capital outflows, the optimal level of reserves recognizing adjustment costs to "sudden stop" capital inflows (for example, when exports suddenly drop relative to imports), the opportunity cost (social cost) of holding reserves, the empirical determinants of reserves, the economic development role of reserves (the so-called, Mercantilist effect), the policy issues related to imperfections in the current behavior of the IMF, and probability models design to handle the optimal level of reserves under uncertainty. This is quite an array of topics and it is not the purpose of this paper to survey this vast literature. For examples of good surveys, see, Calvo, Izquierdo, and Loo-Kung (2012), Obstfeld, Shambaugh, and Taylor (2008, 2010), Moghadam (2010), and Vujanovic (2011). The closets papers found involving international reserves are one by Bar-Ilan, Marion, and Perry (2007) on drift control and one by Costinot, Lorenzoni, and Werning (2011) using dynamic control theory in a macro setting involving tariffs and international capital flows. The former paper addresses several of the topics mentioned earlier. However, it is not a macro control theory model in the context of income and reserves. It is essentially a dynamic simulation model for monetary authorities whose aim is to minimize the expected cost of managing reserves over a long-run time period. The latter paper is in the context of macro control theory and uses familiar control-theory concepts, but it addresses a different problem from the one in the present paper.

The present paper adds a different dimension to the concept of optimal reserves by developing a more general control model that focuses on the trade-off between national income (Y) and international reserves (R). The model motivates the empirical examination of the behavior of the ratio of income to reserves, as indicated earlier. The model in the paper will shed light on the underlying process of reserve accumulation versus income growth and how these two entities are related and behave over time. The empirical examination will show the relevance of the model. The paper abstracts from the specific details in the literature on the motives for holding reserves by using a general utility preference formulation involving Y and R.

In what follows, in the next section, the macro dynamic control theory model is developed in its simplest form to fit the data. The follow section takes up the empirical examination of the Y/R ratio for several developing and mature economics covering the time period 1970 to 2011. The last section has a summary and conclusions.

2. Dynamic-control macro model

Dynamic macro models can become very complex and tedious to interpret very quickly as variables and functions are added. The bare-bones model developed here certainly oversimplifies reality, but this is the price one must pay, if tractability and empirical relevance are desired.

The essence of the model consists of an economy-wide objective function and two differential equations that define the behavior of the state variable and its co-state variable or shadow price. The objective function is an economy-wide utility function which is fairly common in macro models. The utility function is given by U=U(Y(t), R(t)) and consist of the macro income control variable, Y(t), and the state variable, R(t), international reserves. Standard properties for U(.) are assumed. Let i index Y and j index R, so $U_i > 0$, $U_j > 0$, $U_{ii} < 0$, $U_{jj} <0$, and $U_{ij} = U_{ji} > 0$, given continuity. The latter cross-effects will be important in the empirical section.

The stock of reserves, R(t), has four components: gold, special drawing rights, the IMF member's deposit, and foreign exchange holdings by the country's monetary authority . The R(t) is treated as financial capital (in effect, the equivalent of goods and services which affect utility). The basic assumption in this bare-bones model is that the welfare (utility) of the economy depends essentially on Y(t), its GDP, and on R(t), its stock of international reserves. There is a trade-off (substitutability) between Y(t) and R(t), which is the focus of the paper.

The differential equation for the state variable, R(t), is given by the change, dR(t)/dt =R(t)g(S), where g(S) is the rate of change of R(t) and S(t) is $X(Y_w) – M(Y)$, in effect, the current account flow in the balance of payments, where exports depend on world-wide income less imports which depend on the economy's income. The S(t) is a flow variable and just one of the sources of change in the stock of reserves, R(t). For example, the country can always hold more gold, or borrow and deposit more with the IMF. Exports are treated as exogenous. The S(t) is essentially an inverse function of imports, which are a direct function of income. Here, as elsewhere, use is made of the repeated function of a function rule in the derivations to follow.

The canonical Hamiltonian is given by

(1) $H(Y(t),R(t),\mu(t))=e^{-rt}[U(Y(t),R(t))+\mu(t)R(t)g(S(t))]$.

The first-order condition for the control variable, Y(t), is given by

(2) $\partial U/\partial Y - \mu(t)R(t)(\partial g(S)/\partial S)(\partial M/\partial Y) = 0$.

Because of the derivative of g(S) with respective to S and then S with respect to Y, there is a negative sign as in $(\partial g(S)/\partial S)(-\partial M/\partial Y)$, due to exports taken as given in the current account, S(t), so the sign is transferred to the shadow price, $\mu(t)$.

The first-order says that given μ and R at the instant t, the marginal utility of income equals the marginal utility of the reserves offset by the marginal propensity to import, given by $\partial M/\partial Y$, and the rate of growth of reserves, g(.). In other words, along the optimal time path, the marginal utility of income equals the trade-off with the marginal reserves. An increase in Y has both a direct positive marginal utility effect and an indirect inverse marginal utility effect by virtue of the effect of Y on imports M and then reserves through S as explained above.

The canonical form for the co-state shadow price, $\mu(t)$, is given by $de^{-rt}\mu(t)/dy = -\partial H(.)/\partial R$. In differentiation of H(.), the discount factor, e^{-rt}, cancels out. Skipping the details, then, the differential equation is given by

(3) $d\mu(t)/dt = (r - g(S))\mu(t) - \partial U(.)/\partial R$.

The (r – g) is assumed to be positive. Equation (3), then, has a variable coefficient, say, p(t)= (r – g(t)) and a variable marginal utility of reserves. For this nonhomogeneous first-order linear differential equation, the general solution is $\mu(t) = e^{\int p(t)dt}[\ A- \int U_j(t)e^{-\int p(t)dt}dt]$, where U_j is the marginal utility of R and ultimately a function of t. In effect, the change of the shadow price is affected negatively by the marginal utility of reserves, $-\ \partial U(.)/\partial R$. Shifting $-\partial U(.)/\partial R$ to the left side of (3), the now augmented differential equation

$d\mu\hat{}(t)/dt = p(t)\mu\hat{}(t)$, has the much simpler solution, $\mu\hat{}(t) = \mu\hat{}(0)e^{\int p(t)dt}$, the variable rate of growth of p(t) being unspecified. Be this as it may, the term $\partial U(.)/\partial R$ could very well be constant, as we discuss below. As such, (3) then has important implications for the behavior of Y(t) and R(t).

One implication of equation (3) is that the shadow price may increase at an increasing rate of growth. So, along the optimal time path, based on equation (2) and the equation for dR/dt, income, Y(t) and reserves, R(t), will also be increasing at an increasing rate of growth. It is conceivable, of course, that they are growing at the same rate (the Golden Rule), but, this need not necessarily by the case. Much depends on how U(.) is specified.

It is usually assumed that U(.) is homogenous to the first degree. So its marginal utilities are homogeneous to the zero degree. This implies that if Y and R do grow at the same rate, then the marginal utilities are constant. But, if the marginal utility of Y is constant, then, the first-order condition, equation (2), with μ and R increasing, is inconsistent and does not hold. Assuming that equation (2) does hold, then Y and R must be growing at different rates for $\partial U(.)/\partial R$ to be changing and (2) to be valid. To keep within the focus of the paper, the issue then is, “Are the rates the same?” This issue is examined in the next section for a selected number of economies by looking at the behavior over time of the ratio of country income (GDP) to its international reserves (R(t)).

3. Empirical analysis

To summarize briefly before proceeding, the dynamic macro control theory produced a first-order condition on the control variable national income (Y) for given values of the state variable international reserves (R) and its co-state variable the shadow price (μ) at a point in time. From the analysis of the first-order condition and the other variables, given the assumptions, Y(t) and R(t) had to grow at different rates over time. This outcome is due to the design of the model and it rationalizes the empirical behavior of the ratio Y/R, as will be shown for a selected number of countries and over the time period 1970-2011.

The country data on income and reserves are from the IMF. The reserves are in current US dollars. The income data are in national currency which was converted to US dollars by the author using year-end exchange rates contained in the data set. Both variables are in nominal values but since a ratio Y/R is used, no real value conversion is needed. The countries selected are in two groups, developing economies (Indonesia, S. Korea, Thailand, Singapore, Malaysia, the Philippines, and China—the order follows the order in the Figure 1) and mature economies (Japan, Germany, and the U.S.A.).

Figure 1 shows the time path of the Y/R ratio for the developing economies. While there exists volatility in the seven time series, since around 1982-1985 the trend is decidedly downward due to the fact that R is growing faster than Y. To give a sense of the magnitudes of the ratios observed, by order of appearance in the Figure from the early period to the later period, the ratios of Y/R range (early/ later) from about 55/5, 56/3, 24/2, 2/1, 9/2, 45/3, and 107/2 for China.

On the other hand, for the mature economies, Figure 2, with the exception of Japan the trend is decidedly upwards. For Japan, the time series trend is downwards. The ratios range (from the early to the later period) from 59/5 for Japan, 11/72 for

Germany and 72/190 for the USA. These empirical results for both sets of countries are consistent with those shown by Vujanovioc (2011).

What is the economic significance of these results? For the developing economies, the results mean that they put relatively greater utility (welfare) value on reserves compared to national income. While, as indicated earlier, reserves can be thought of as equivalent to real goods and services, there is a decided difference in terms of the distribution and ownership of the two entities. The government's monetary authority owns and controls the reserves. The national income is distributed among the people and mostly owned and controlled by them. Do the people benefit from this distinction? In fact, Alfaro and Kanczuk (2006) argue in their stochastic model that the optimal policy is to hold no reserves. Others, like Calvo, Izquierdo, and Loo-Kung (2012) take the position that reserves are needed as insurance against sudden falls in exports.

Ultimately, the rational for the government to accumulate reserves in a world economy where uncertainty in economic trade and capital movements exists is to manage its risk exposure in such a world economy. Trading off income for reserves in the present dynamic macro model was shown to be an optimal outcome in terms of maximizing the economy-wide utility (preference) function. And, the empirical facts fit the model. While the simple model developed here does not resolve the real-world issue of why hold reserves, nevertheless, it offers a theoretical basis for the empirical facts.

Why do mature economies typically hold relatively less reserves? In the context of the dynamic model developed here, the answer is in terms of their relative preferences for income versus reserves. Outside of the context of the model, the argument is over absolute size versus relative size. While the mature economies hold a smaller percentage of reserves to income, since income is so much larger for mature economies compared to developing economies, the absolute amount of reserves is quite large. In risk-management, the absolute amount of reserves (protection) is what counts.

4. Summary and conclusions

The macro dynamic behavior of national income and international reserves was model by a simple, bare-bones control theory model. Fairly standard assumptions were made. The theoretical outcome in terms of the control variable, national income, and the state variable international reserves was that while both may be growing they both have to grow at different rates. Since the model demands different rates to meet the equilibrium condition, the ratio of income to reserves will be changing. The model does not address directly the policy issue of why the ratios behave differently for different types of economies (mature versus developing). Implicitly, the explanation is embedded in the country-wide utility function. The empirical results, however, fit the model and do show that, indeed, developing economies have falling ratios and mature economies have generally rising ratios. The literature, using different models, addresses the policy issue of the optimal level of reserves. The present paper looked at the trade-off between national income and international reserves, using a distinctly different model.

The welfare implications of the rapid accumulation of reserves for developing economies were discussed briefly. The discussion showed that, in effect, income is being sacrificed for reserves for the purpose of risk management. It is a difficult choice to make—utility today from income versus utility tomorrow from reserves. In many developing economies, the choice may be starving today versus eating today. In some sense, the literature is correct in arguing that the optimal level of reserves is zero. In other words, although outside of the context of the model used here, reserves are a waste of resources. There are other ways to solve the availability-of-credit problems, ways that, of course, do exist but could be improved upon as the literature shows.

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Figures:

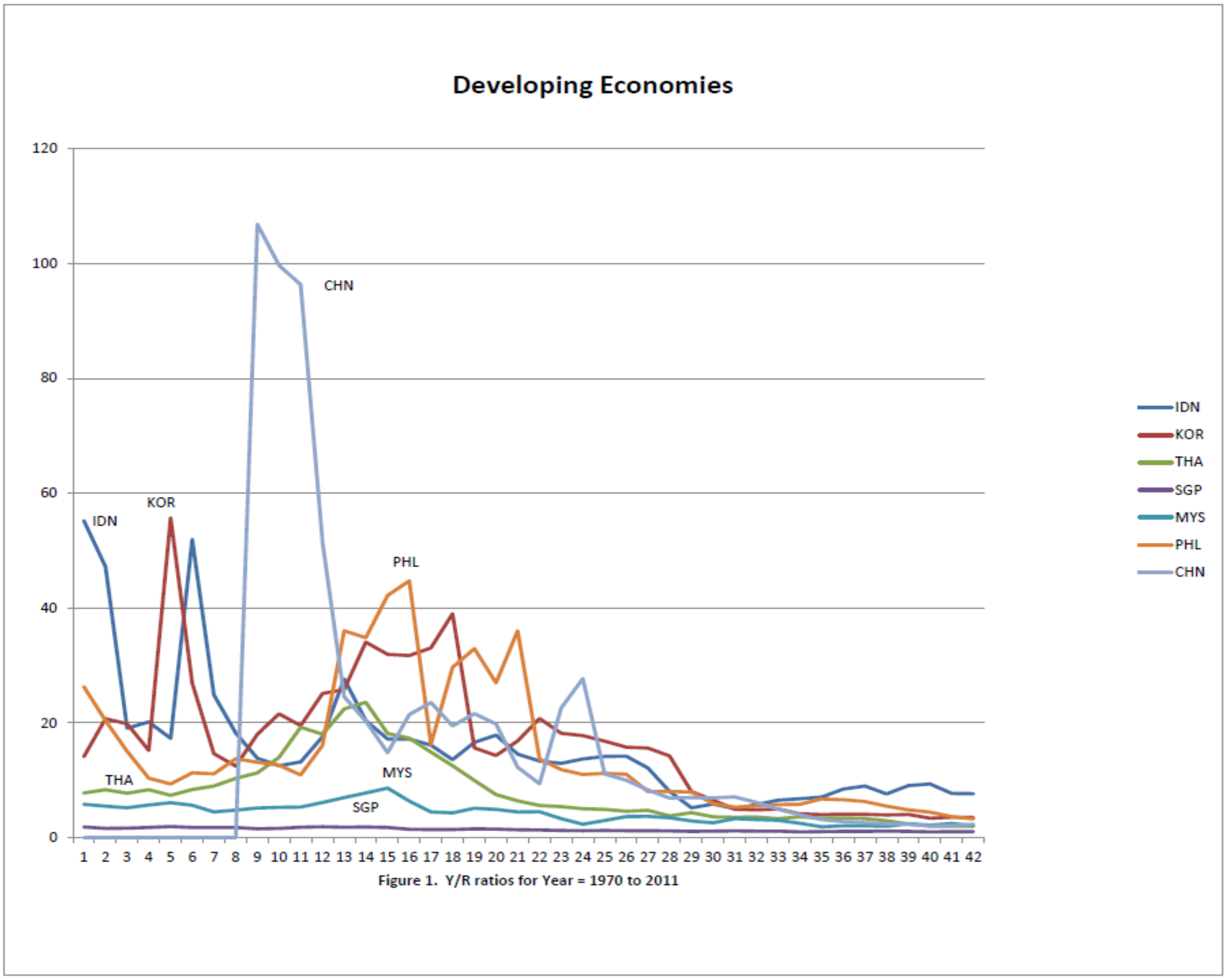

Figure 1. Y/R ratios for Year = 1970 to 2011

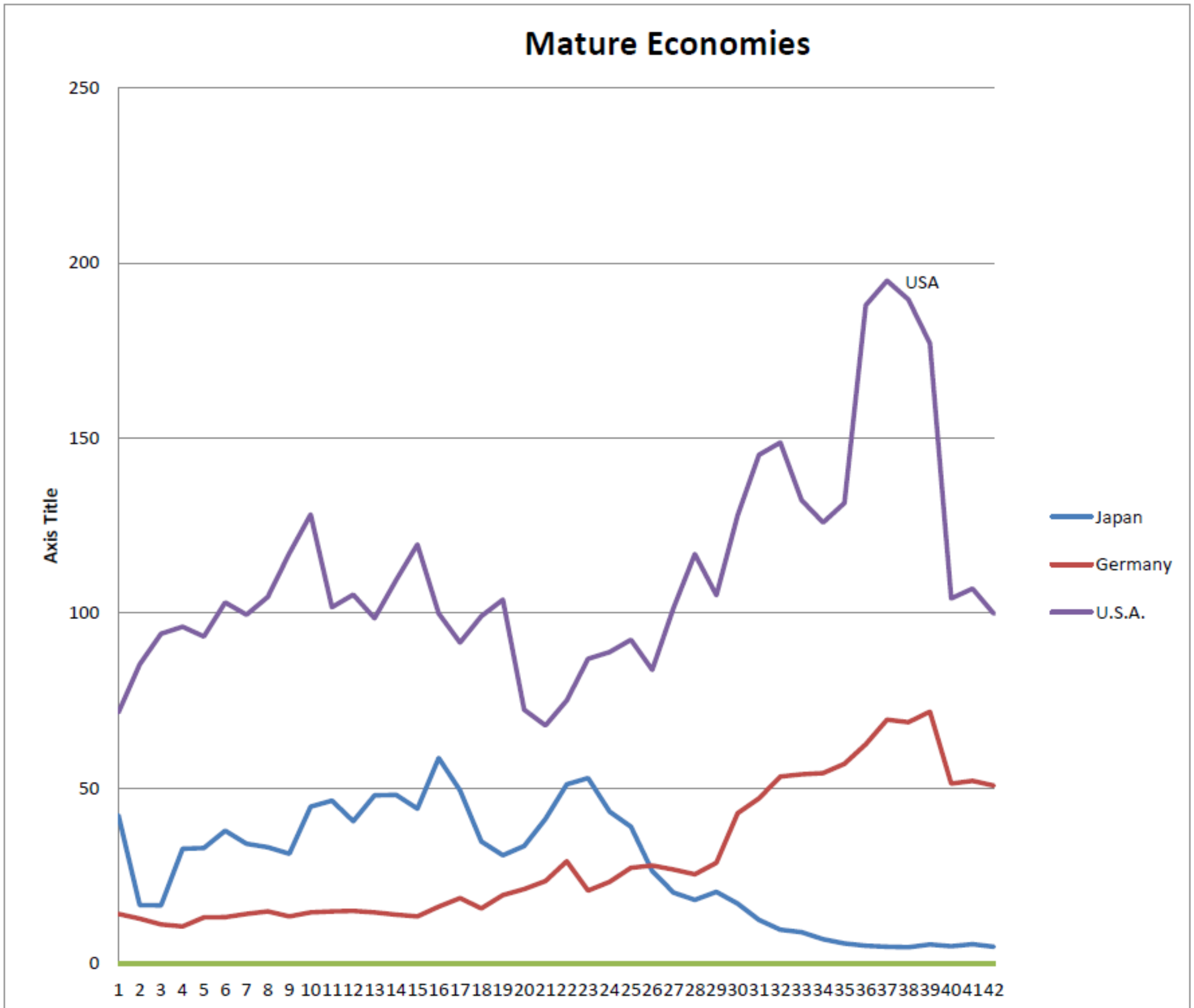

Figure 2. Y/R ratios for year = 1970 to 2011