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Abstract

The paper offers an alternative approach to analyzing stock market time series data. The purpose is to develop descriptive, more intuitive, and closer to reality analogs of the behavior of US stock market prices, as indexed by the S&P500 stock price index covering the period October 2003 to October 2008. One analog developed is the "escalator principle" and the blind man. The approach is to treat prices as a random and independent variable and use extreme value theory to judge probabilistically whether prices and their attributes are from an initial universe or whether there has been a regime change. The attributes include the level, first difference, second difference and third difference of the ordered price series. Various graphing tools are used, such as, probability paper and different specifications of exponential functions representing cumulative probability distributions. The argument is that traditional time-series analysis implies a given universe, usually normal with either a constant or time-dependent variance (or measureable risk) and consequently does not handle well uncertainty (non-measureable risk) due to regime changes. The analogs show the investor how to determine when a regime change has likely occurred.

Keywords: S&P500, Probability, Regime, Uncertainty
JEL Classification: C19; C22; C49; G10

Introduction:

Traditional time-series analysis and all its mathematical forms have a voluminous literature. Much of the econometric essence of this literature can be obtained from Enders (2004). Another part of this literature is finance papers generated by the now classic paper by Fama (1965) on security price behavior and its interaction with its expected price behavior by Beja and Goldman (1980). A considerable part of this literature contains finance papers motivated by the Black-Scholes model (1973). The distinguishing and obvious characteristic of the price data used in all the analysis is that it is used in its natural time-ordered form.

Our purpose here is not to try and compete with this literature and its varied theoretical and empirical approaches. Our purpose is to make a fresh start and to develop descriptive, more intuitive, and closer to reality analogs of the behavior of the US stock market, as indexed by the S&P500 stock price index and as seen through the eyes of the investor. Our analysis is based on a re-ordering by value of the price data. Natural time-order price data initially will be used to describe the price behavior, but since our concern is with a probability test or judgment as to whether a given price series is from an initial universe or from a different regime, the price series must be re-ordered and treated as random events which are then applied to a cumulative probability distribution function. Monthly data will be used for the period October 2003 to October 2008 to illustrate our analysis.

The main analysis consists of using extreme value theory with the re-ordered price series. What will be detailed later as the "escalator principle" will be developed as an analog to understanding the behavior of stock market prices. The analysis will bring out the significance of the first, second, and third differences in the re-ordered price time series, p(t). In other words, the practical investor has a choice of either analyzing the naturally ordered prices as is traditionally done or analyzing the re-ordered prices. With the re-ordered prices and a given cumulative distribution function (CDF), a test of universe or regime change can be made.

In what follows, the second section is on extreme value theory as a tool to analyze the stock market price behavior. Then, in the third section we examine the first, second, and third price differences in the context of the "escalator principle." The fourth section contains a summary and conclusions.

<u>Extreme Value Theory</u>:

The basic assumptions behind extreme value theory are that the parameters of the initial universe or probability distribution remain constant or if parameter-changes have occurred, then such changes can be adjusted, and the price draws or events are random and independent of each other and thus over time. This last assumption can be discussed later, but for now we accept it. It is necessary in order to plot points on probability paper.

Extreme value theory shows that as the sample size increases, the range of values for a given event also increases and its probability need not be particularly small. If the event is an extreme value (say, more than three sigma from its mean), then the probability of it could be relatively high. For example, if the daily high and low S&P500 re-ordered values are plotted on probability paper (paper that is specially scaled to show the cumulative probability as a function of the price), using either the normal curve or a general exponential function, as the price index rises, under normal market conditions a linear function is obtained. This function represents the theoretically given universe for the sample. If an event occurs whose value is off the linear function, thus giving a higher (or lower) probability than normal, the event will still be considered as coming from the initial universe or regime as the previous events, as long as the deviation is not too great. In other words, even though there is a blip in the probability tail for an extreme value, if the blip is not too large, the event is considered as coming from the initial universe.

But, if the event is way off the linear function, then it is considered as coming from a different universe. A regime change has occurred.

In his classic paper containing four lectures on extreme value theory, Gumbel (1954), applying the theory to the US stock market over the period 1897-1947, with the Dow-Jones index, stated:

"If the straight line is accepted, the conclusion may be drawn that the crash at the end of the twenties really spelled the doom for the system then prevailing, and that the present phase of the capitalist system in this country is fundamentally different from the previous one," (p. 46)

To elaborate further, take samples from a presumably normal card deck with replacement. Say, after 52 draws you get four aces. This is what you would expect to get from a normal deck. But, say you obtain after 520 draws, 260 aces, would you still consider the deck to be normal?

On the one hand, extreme value theory says that if a very large sample is taken, extreme values are likely to show up, since the presumption is that the universe of events or values thereof is unlimited. The fallacy of the 3-sigma rule contradicts this by saying any value more than 3-sigmas out is inconsequential. On the other hand, if the frequency of the extreme value is too high, it is unlikely that the deck (in this example) is normal.

In some of the literature, a probability density function (pdf) with an extreme value with a relatively high probability is referred to as a probability distribution with a "fat tail" (see, Taleb, 2007, for an historical survey, although Taleb does not explicitly use the word "fat" but uses the phrase "long tail" or extremism). The notion of a "fat tail" owes its origin to Mandelbrot's work (1963, 1972) on the behavior of cotton prices. Fama's paper (1965) used the "fat tail" concept and motivated many studies on security prices. Although it has been argued by some that too much is being made of the concept that has been obvious to investors and researchers for a long time.

Related to the "fat tail" concept is the extreme value theory. It has been around a long time (at least since 1922) in the geology and flood control literature (see, Gumbel, 1954, for his list of references). What complicates the use of extreme value theory in the case of the stock market is the fact that the stock market is part of a global financial system. The theory as used by geologists only has to contend with a given river (say, the Mississippi river), and not the whole world (the butterfly effect, not with standing).

The issue underlying our use of the extreme value concept (and, for that matter, the "fat tail" concept) is to differentiate between Knight's (1964, Chapters 7 and 8, but the first edition was 1921) measureable risk and non-measureable risk or uncertainty. Keynes also had this differentiation in mind (see, Keynes, 1936, Chapters 12, 13, and 22). Others use the Black Swan concept to describe the uncertainty of unknown events (See, for example, Taleb, 2007).

Measurable risk is based on an objective probability distribution, such as used in life insurance and fire insurance. As a person's age increases (sample size), the probability of dying by age 110 approaches one (assuming falsely that 110 is a finite limit). If houses are burning down more frequently than normal, a regime change is suspected.

In the case of stock prices, in normal times, small deviations +/- from the linear function on probability paper would be considered measureable risk (of course, rating agencies come into play here, but we discuss this later). Even if the deviations were relatively large, in other words, extreme values, some researchers would still consider the risk measureable. In such a case, insurance against loss would be available (like credit default swaps). However, if the deviations were too large, there is no objective probability and no measureable risk. In this case, the extreme value is from a different universe or regime, so its occurrence is based on subjective probability and it is an uncertain event. In such a case, insurance is not available. The investor will self-insure by hedging.

The issue then is how do we measure the risk of a given stock, when its value (price) is uncertain in the Knight-Keynes sense? As an aside, this is the problem today faced by investors considering investing in debt securities that are backed by sub-prime mortgages, the so called collateralized debt obligations (CDO's). This same problem is also faced by the US Treasury, now that the Emergency Economic Stabilization Act of 2008 has been signed into law (October 3, 2008).

In practice, the amount of risk associated with a given stock for a given period is determined by rating agencies like Value Line, which gives the Beta coefficient for a particular firm, a measure of the quantity of risk relative to the market basket as a whole (usually, the S&P 500). Numerous other methods of determining the amount of risk exist.[1] In any case, on the presumption of a competitive market with full information,

[1]Couriously, the Black-Scholes model (1973) with all the intrigues associated with its use of a partial second-order differential equation still only relies on σ^2 and the normal CDF to measure risk.

financial researchers and some economists argue that the market will perform efficiently in the sense of determining the true price and amount of risk (if such were the case, why does the price change?). Hence, we have the expression, a fair market value. In normal times, this risk can be considered a measureable risk in the Knight and Keynes sense. But, with uncertainty, we are investing in abnormal times. It is like betting on drawing an ace from an unknown deck. This is where extreme value theory comes in. The problem is that you cannot test the deck ahead of time for extreme values. At least in a game of chance, you can demand to see the deck and shuffle it, spread it out face up, to see if it is normal.

The problem borders on determining the appropriate sample size. Should we analyze the last ten years or the last 20 years or longer? The longer the sample period, the greater is the likelihood of capturing many different universes or regimes, all of which may be quite different in fundamental ways. Yet, if a very long time period gives observations that are extreme but close to the linear function, then extreme value theory says the observations are from the initial universe. Hedging by having a portfolio that is composed of a mixture of risky stocks can help in some cases of uncertainty. But, when the whole stock market is falling, hedging does not help. Fire insurance or flood insurance work when only a few homes are affected, but if the whole community is affected, then a larger risk pool is needed. The insurer of last resort becomes the government.

In Figure 1a, we show the high and low of the naturally ordered S&P500 price index since 1950, daily, to 9/25/08. Up until 4/15/94, the index rises exponentially at a fairly constant and low rate. Then, it rises at a very high rate until it peaks around July or August 2001. Of course, after 9/11/01, it falls very rapidly to about 5/20/03, then it peaks again in June 2007. It has fallen ever since. Clearly, a new regime has occurred beginning 4/15/94. But, there have been three other regimes since then.

Can one determine when another extreme value (high or low) will occur and its probability from the existing data? It can be argued that prior to 4/15/94, there is not sufficient variance in the data to form an opinion about the large variance (or risk) that ultimately followed. In other words based on the pre-1994 data, the post-1994 data with its high variance could not have been predicted. The extreme value theory would show that the post-1994 data did not come in all likelihood from the previous regime or universe. Figure 1b shows the observed or naturally ordered time path for p(t) from October 2003 to October 2008. The rapid rise to the peak at the 49th month (October, 2007) and the subsequent fall are evident.

For the main analysis of the re-ordered stock market prices using probability paper, we use a shorter period of time as indicated above. Figure 2a represents the plots on probability paper where the cumulative probabilities of random events are plotted on the specially scaled Y-axis and on the X-axis are plotted the re-ordered values of the stock price indexes using a linear scale. For now, we follow Gumbel's (1954) cumulative probability function, where $F(p) = \exp(-e^{-y})$ and $y = (p - u)\alpha$. The double-log transformation of F(p) yields $y = -\ln(-\ln F(p)) = \alpha p(t) - \alpha u$, a linear function of p(t). The probability paper is ruled for y on the Y-axis and the re-ordered p(t) on the X-axis. Alternatively, y and p(t) can be plotted on arithmetically scaled paper as shown in Figure 2b. The upper right five or six plots deviate from the linear graph given by R and do not appear to have come from the initial universe. “Fat tail” followers would argue the contrary. The cumulative probability appears to rise too fast as it approaches one (y = 4.1190 or CDF = .984). The important point to note is that the Gumbel double-log function is a linear function in the re-ordered p(t). Later, we will use a Weibull (See Web site) function which is linear in the log of re-ordered p(t).

Briefly, for now the plotting process on probability paper goes like this. The original S&P price index is re-ordered from low to high. The original monthly rank corresponding to the naturally ordered monthly price index is now not in a continuous sequence, since for any one month, the price could be high or low. So, a new monthly time index series is constructed from 1 to 61, the sample size, in alignment with the re-ordered price values. We will for simplicity still use the notation p(t) for the re-ordered values. The re-ordered prices will be the basis of the main probability analysis throughout the paper, except for the initial descriptions that are based on the naturally ordered prices.

The next requirement is the cumulative probability for the random price event. It is approximated for a exponential function by the formula, $F(t) \sim (t - .3)/(n + .4)$, where t is the new monthly time index corresponding to the new price array (other formulae can be used such as $F(t) = t/1 + n$ (used by Gumbel, 1954), or $F(t) = 1 - \exp(-\lambda t)$, where λ is a parameter (available on XLS)). Then, on probability paper, the cumulative probability and the re-ordered price index are plotted.

The best-fit linear functions (determined here by eye sight) are shown in Figure 2a as R1, R2, and R3. As indicated, one can argue that there appear to be three regime universes given by the respective R's. R1 is for relatively low stock prices. R2 is for middle-range stock prices, and R3 is for high prices. It is difficult to argue that all these ranges of prices come from the same, single universe. Each linear function represents a particular CDF, where Pr(p =< p*) is implicit. The plots represented by R2 are not likely outcomes from R1. Similarly, the plots represented by R3 are not likely outcomes from R1 or R2. The respective deviations are too large.

For example, a plot at the lower end of R2 with a CDF value of 40 percent has about a 55 percent chance of coming from R1, a 15 percent error. A plot on the upper part of R3 has a 99.9 percent chance of coming from R3, but no chance of coming from R1. Or, from the reverse perspective, the lower-valued plots from prices in the 1050 to 1230 range have a 1 to 50 percent chance of coming from R1, but a 40 to 55 percent chance of coming from R3, an error of 15 percent. So, the actual probabilities of the random price events cannot be attributed to outcomes from a single universe.

The variance of the price points represented by R1 is not sufficient to predict the variance of the points represented by R2. And, the variance of the price points represented by R2 is not sufficient to predict the points represented by R3. As we argued before, risk information is in the variance. And, there simply is not enough information to have predicted the extreme values as coming from the initial universe. So, the notion of a "fat tail" as it relates to Figure 2a or 2b does not apply. Or rather, there is a fat tail but it is not from the initial universe.

Another way to analyze the monthly stock market price behavior is to use linear scales by transforming F(p) and p(t) into logs. We use a general Weibull exponential function given by $F(p) = 1 - \exp\text{-}(\lambda p)^{\beta} = \Pr(p =< p^*)$, with constants λ and β. The transforming of F(p) to a double-log function yields $\ln(\ln(1/(1 - F(p)))) = y = \beta\ln\lambda + \beta\ln p(t)$, a linear function in logs (note, again, Gumbel is linear in the absolutes). We can now plot y against lnp(t) using linearly scaled units.

To be consistent with the previous approximation of F(p) used for Figure 2a and 2b, we substitute for F(p) the approximation, $F(p) \sim F(t) = (t - .3)/(n+.4)$, shown earlier. Then, the double-log calculations are made using F(t) instead of F(p). Figure 3a shows the graph. Approximate linear equations are indicated by R1 and R1'. In either case, between 7.1 and 7.2 logpriceX values, the deviations appear to be too large to have come from the initial universe or regime.

Using another approximation, let $F(t) = 1 - \exp\text{-}\lambda t$, from XLS ($\lambda = .0323$). The cumulative probabilities in double-log scaling are on the Y-axis and the logpriceX is on the X-axis (see Figure 3b). The R1 and R1' approximate-fit linear functions, suggest that the deviations are too large to have come from the initial universe. Note, as indicated earlier, to be consistent both figures are based on using the time rank order (1 to 61) in F(t) as a proxy for the absolute price rank order (1053.8 to 1576.09) in F(p).

What these results suggest is that we need to look at higher moments or something akin to moments to see if there is enough variance information to predict the extreme values. Strictly speaking, the kth moment (about zero or the mean) is a parameter, a single value as in $\Sigma x^k f(x) = M_k$. For our purposes, it is more informative in terms of variability to see the actual deviations of a series from point to point in time.

First and Higher-Order Price Differences and the "escalator principle:"

We begin by examining the naturally ordered price graph of the first differences against time as in $d(t) = p(t) - p(t-1) <> 0$, where p(t) is the naturally ordered S&P monthly stock price index. In effect, the d(t) is akin to the single arm of the first moment as in $[p(t) - p(t-1)] = x^{k=1}$. But, before we examine the first differences in p(t), we give the details of the "escalator principle," developed as an analog for understanding the price behavior.

As an analog to the price behavior we try to model, consider a blind man starting up a typical electric escalator of steps. The behavior of the step rises is by design given and fixed and represents analogously the given universe. The issue or hypotheses is to test whether the escalator is the same universe or not by analyzing the step rises. For the given escalator, initially, the steps rise slowly and then the rise remains constant for a time. Then, as the step rise gets smaller and smaller, the blind man senses that the escalator is approaching the top of the stairs and when the step rise disappears altogether, he knows the escalator is at the top and it is time for him to get off.

The rise itself can be thought of as analogous to the first difference of the re-ordered price series. But, the second and third differences are also significant indicators to consider. As we discuss in a moment, we assume that the typical investor is blind to the path of p(t), unless he examines the first, second, and third differences. These falling differences signal when the top of p(t) is approaching (and, in an analogous way when the bottom is approaching). It is perhaps a bit of a stretch to assume investors are "blind" to p(t), like the blind man is blind to the escalator, but seeing is not always believing. The investor may in some sense see the price level, p(t), rising but unless he is keenly aware of the price differences, he is for all intents and purposes blind. The blind man survives because he is sensitive to the differences. The investor must do likewise.

Figure 4 shows the first differences of p(t) naturally ordered for the period October 2003 to October 2008 for 61 observations (less one for the one lag). The general pattern from left to right of the series consists of several positive d(t)'s, then one or two negative d(t)'s. As the 49^{th} month (October, 2007) is approached, however, the sequence is reversed, one or two positive d(t)'s, but several negative d(t)'s. In other words, as the market crashes (our data set only goes to October 4, 2008, but the crash was well on its way), the differences are more frequently negative, as expected when the trend changes from expansion to contraction. The message given by the pattern is that when first differences start to become more frequently negative than positive, we should soon expect the market to fall significantly. The smart investor will sell short and buy back when the market has fallen. Our concern, however, is with testing using extreme value theory whether the first differences are from the initial universe or has there been a regime change.

To this concern, the next task is to re-order the price series and then to order the first-differences of this series from low to high. By construction, all the differences will be positive and are assumed to be random. Then, using the previous method to approximate the probabilities, plot the probabilities on probability paper (PP) with the first-differences of the re-ordered p(t) on the X-axis following the Gumbel linear model (Figure 5). Since the first-differences are ordered, we could as an alternative to them use the time-order index (1 to 60, now) on the X-axis. In any case, plotting F(t) = (t-.3)/(t + .4) ~ F(Δp) against Δp on probability paper will give the cumulative probability of the differences and whether they appear to come from the same universe or not. In other words, for the random collection of first-differences to make any sense, the investor has to decide whether they come from the given universe or another universe.

Similar to before, using the Gumbel model (where for now we use F(Δp) ~ t/61) for the double-log scaling of y against the linear Δp(t), see Figure 6a, at about a first difference equal to 10 (July, 2007), the deviations appear to be significantly large, suggesting a regime change. Again, the "fat tail" followers would argue otherwise. Using the Weibull model (with F(Δp) ~ t-.3/61.4), see Figure 6b, the double-log y is plotted against the log of Δp(t). Here, there appears to be two tails, a lower deviation from the linear equation R1 and an upper deviation from R1. This suggests two regime changes from the original regime.

To use our escalator analogy and to be consistent with the previous analysis, the step rises are treated as random events, all positive, and ordered from low to high. The blind man has to believe that as the first differences (the step risings) are getting smaller, the cumulative probability for a given step rise must be approaching one as the step rise approaches zero. This belief is necessary to judge that the given escalator has not changed. In other words, do falling step rises indicate that he is on the given escalator? To do the probability test, we need to read the probability function in reverse, so Pr(ΔH ≥ ΔH*) = 1 – F(ΔH), where ΔH is the step rise. The chance of the rise getting smaller has to be increasing for him to believe that the top of the stairs is approaching (meaning he is on the same escalator or regime). If, for example, there is only a 40 percent chance that the given step rise is falling, a 60 percent chance for the inverse, the step rise is rising, he will be uncertain about whether he is approaching the top or not, or equivalently, whether the escalator is the initial one. A similar analog can be developed using an elevator and the rate at which it is approaching a given floor.

More information can be gleamed from the price series by looking at the second differences, $d(t)2 = [p(t) – p(t-1)] – [p(t-2) – p(t-1)]$, as shown in Figure 7 for the naturally ordered p(t). Here, the pattern generally is as follows: the width in time between high d(t)2's (>0) and low d(t)2 (<0) values is fairly consistent (two to four months of negative d(t)2's) until about the 42nd month (March, 2007), then the high-low values increase rapidly and the width (or spread between fluctuations) gets smaller. The message is that once the time width between high and low d(t)2's starts to shorten, high volatility or variability can be expected. The market will soon become very unstable. The smart investor can play this volatility by selling short when the d(t)2 is high and plan soon to buy back the stock when the market is low to cover the short sale.

To test for a change in regime, Figure 8 shows the plots for the cumulative probabilities obtained from the earlier approximation, where $F(t) \sim (t -.3)/59.4$, against the 2nd differences of p(t) re-ordered to use probability paper. Treating the 2nd differences as random outcomes, the straight line in the figure represents where the plots from the initial universe are most likely to fall. The upper right plots are presumably from a different universe. So, when large positive second differences occur, it is likely that they are from a different universe, "fat tail" followers not withstanding.

This conclusion is also evident when the double-log linear Gumbel y (where $F(\Delta 2p) \sim t/60$) is plotted against the ordered 2nd differences on a linear scale as shown in

Figure 9a. The above conclusion also appears in Figure 9b, using the Weibull model which is linear in logs. There appears to be two regimes other than the initial regime (represented by the linear function R), one for small 2nd differences and one for large 2nd differences. Wide swings in the second differences tell the investor it is time to sell out. When the blind man senses improbable second differences (analogous to the change in the step rises), either the small second differences have too low a cumulative probability or the large second differences have too large a cumulative probability, both relative to the initial regime, he suspects that there is something wrong with the path of the escalator and that there is an uncertainty as to whether it is heading for the next floor. Equivalently, he suspects that there has been a regime change.

Figure 10 shows the third differences for the naturally ordered p(t) against time for the same period as before (allowing for the lags). From about the 42nd month (March, 2007) onwards the high and low difference values get further and further apart, forming a funnel shape from month 42. Previous to month 42, the third differences fluctuated fairly consistently between +/- 100 around zero. The message is that once the third differences start to increase in value, volatility has set in, so the highs can be expected to be followed by the lows. As before, the smart investor can play on this variability by selling short at the high difference and buying back at the low difference.

In Figure 11, the cumulative probabilities approximated by F(t) ~ (t - .3)/58.4 are plotted on the specially scaled Y-axis along with the ordered positive third differences on the log-scaled X-axis on probability paper. The log scale had to be used because of the way the third-difference values occur. The values are virtually zero up to the 16th observation and near zero from 17 to 47. From observation 48, the third difference goes from .1 (with a CDF of .81) to 12.04 (with a CDF of .98). In spite of the graphing difficulty, treating the third differences as random draws, it appears that the upper right hand positive plots are from a different universe compared to the left-side plots close to the Y-axis.

A clearer graphing is given in Figure 12a. It shows the plots for the Gumbel model using y against the ordered linear scale for the third differences. The upper right side plots appear to deviate too far from the initial universe represented by the linear function given by R. To put it another way, the cumulative probabilities are too low for the small third differences and too large for the large differences, compared to the initial universe given by the plots on R. Figure 12b shows the plotting using the Weibull log mode. The upper and lower tails are similar to those in Figure 12a. For either figure, the message is that large positive third differences and small third differences are most likely from different universes than from that for the central plots represented by R. Rather than refer to these extreme results as evidence of "fat tails," we argue like earlier that these results indicate regime changes. The fat tail thesis implies that the variance or risk is a constant and measureable, whereas the regime change argument implies that the risk has changed, even though its measurement is uncertain.

By way of the escalator analog, small and large positive third differences represent changes in the rate (second differences) at which the step rises are changing. Improbable third differences or rather unlikely third differences cause the blind man to have uncertainty about how the escalator is behaving and whether it is heading for the top floor or not.

<u>Summary and Conclusions</u>:

The purpose of this analysis is to put forth analogs of the behavior of random stock market prices as given by the S&P's 500 index. I described an intuitive and realistic methodology to gain an understanding of the non-measureable risk in the random behavior of stock prices.

The point of my analysis is that there is information in what is equivalent to the first, second, or third differences of the re-ordered random price series p(t).[2] It may even be useful to go to higher order differences, although that does not appear to be the case. In other words, buried in the differences is risk information about the different universes or regimes of the random price series, p(t).

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[2]Enders(2004, p. 7) argues, "…that third and higher-order differences are never used in applied work." My guess is that this may be so, because third and higher-order differences present a problem when trying to find the characteristic roots for the higher-order differences to obtain a specific solution or function, p(t). My task is not to try and find a solution function, per se. The task is to re-order the actual price series and when the ordered first, second, and third differences start to behave as described in the text, then it is time to sell out or sell short. In other words, we are not trying to find a suitable forecasting function which reflects or is a summary of all past price-differences of the actual price series. We want to analyze the behavior of individual re-ordered prices and price differences and on the basis of extreme value theory come to some judgment about whether the price series comes from the initial universe or from another universe.

There may appear to be some resemblance of our difference analysis to the Black-Scholes model (1973), but such appearance is deceiving. The partial differential equation

(PDE) of the Black-Scholes model (see, for a rigorous mathematical treatment, Stampfli and Goodman, 2001) is essentially in spirit a weighted average of the first and second derivatives of a well-defined function, say, f(t), where the weights involve the mean and the variance of the normal CDF. Where my analysis differs is that the random price differences are not derivatives of a given function, but are more like incremental changes which suggest or imply different universes or regimes and not a given universe. As I tried to show and argue, an awareness of the ordered price differences and their resultant cumulative probabilities can signal a regime change. Of course, being aware of the differences or being sensitive to them does not guarantee that a given investor will act on them. While I argue that variances and risk information will vary with regimes, I am not trying to develop here a mathematical model. As stated at the outset, I seek analogs that are realistic and intuitive to the typical investor. The practical investor can easily re-order the actual price series and plot the first, second, and third ordered differences of the re-ordered price series and come to some judgment and conclusion about what decision to make concerning his investment. It is also hoped that such analogs will be helpful to financial and economic policy makers.

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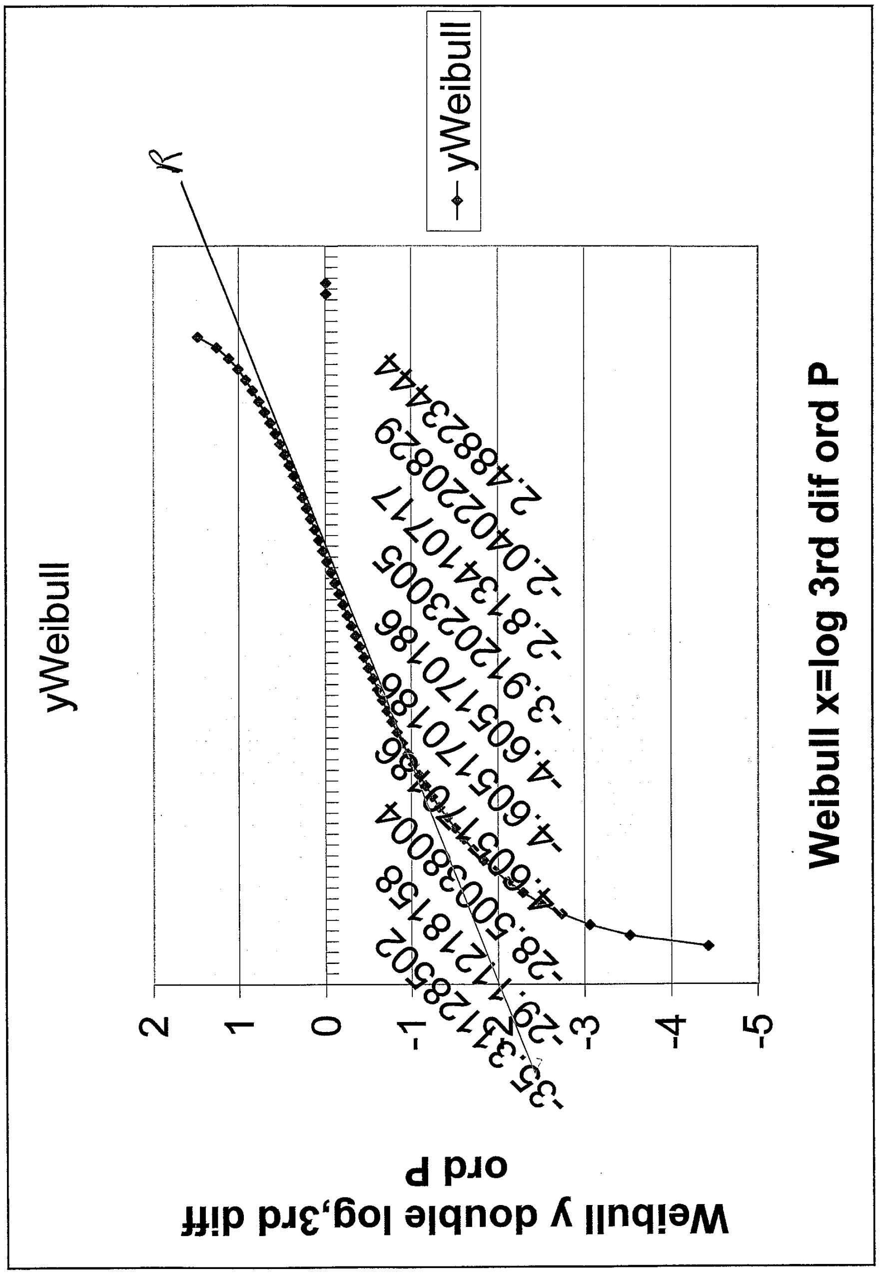
Fig 12b

1/19/09



12b

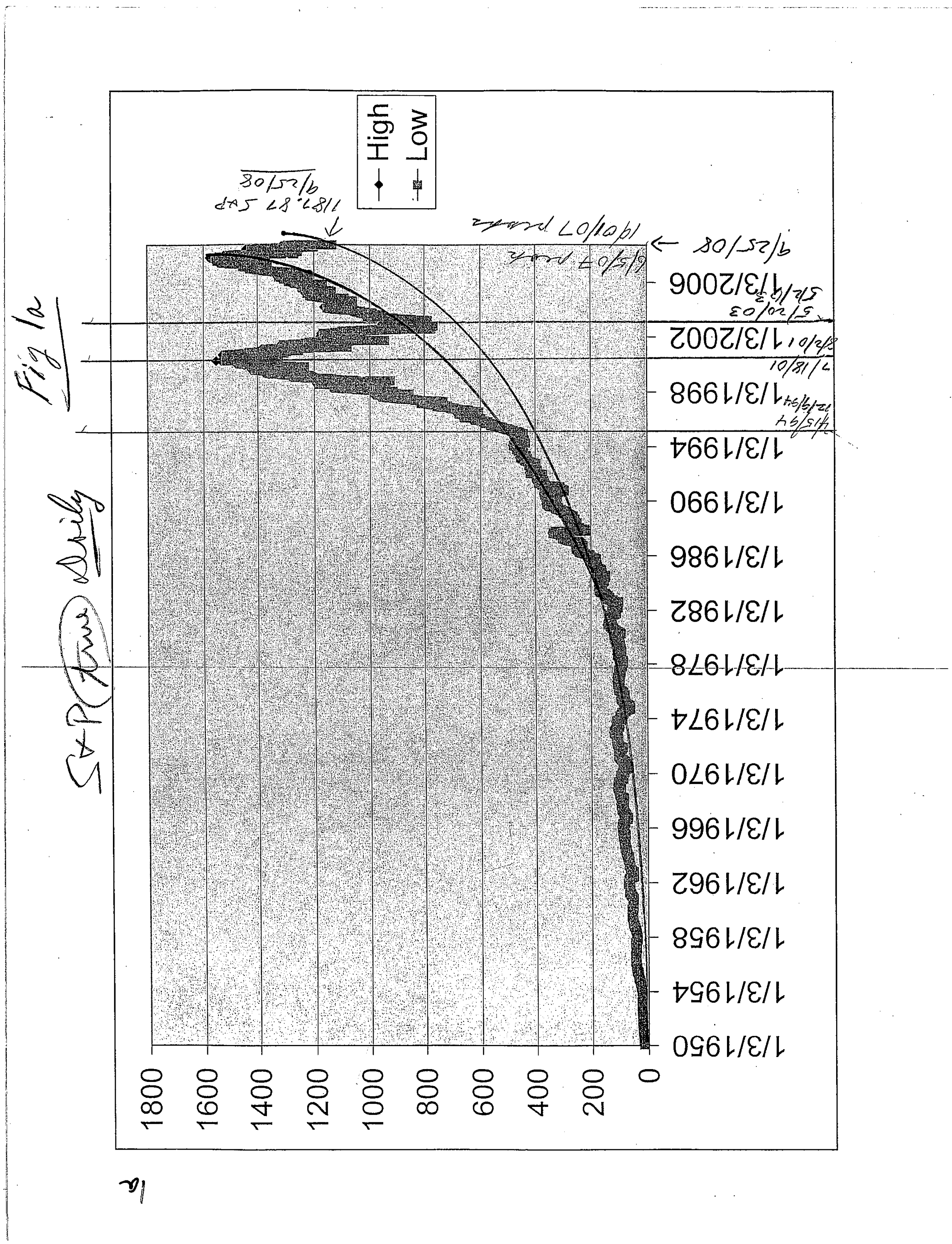
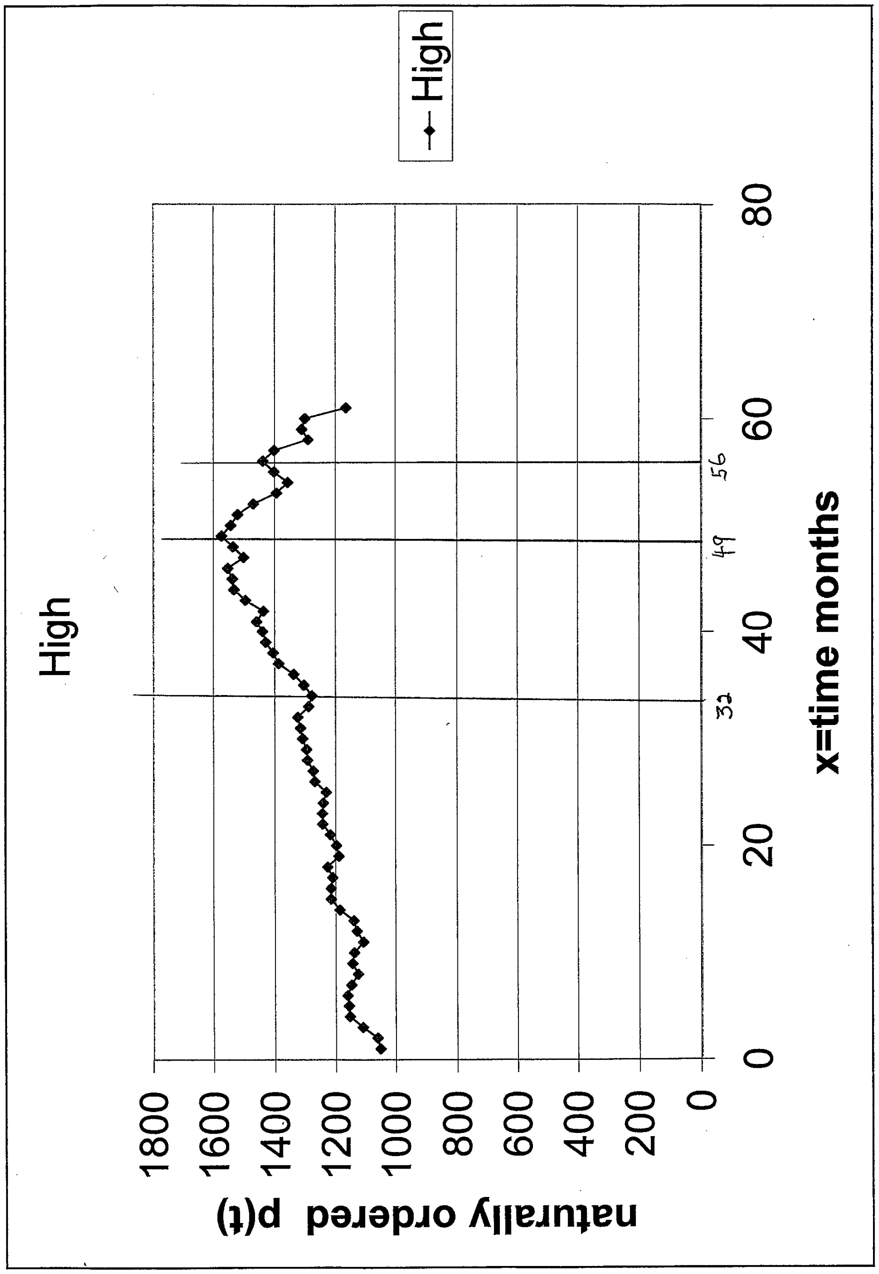

High
Low
1800
1600
1400
1200
1000
800
600
400
200
0
1/3/1950
1/3/1954
1/3/1958
1/3/1962
1/3/1966
1/3/1970
1/3/1974
1/3/1978
1/3/1982
1/3/1986
1/3/1990
1/3/1994
1/3/1998
1/3/2002
1/3/2006
Fig 1a
S+P Daily
9/25/08
1187.87 S&P
10/10/07 peak

Fig 16


High
naturally ordered p(t)
1800
1600
1400
1200
1000
800
600
400
200
0
0
20
32
40
49
56
60
80
x=time months
High

16

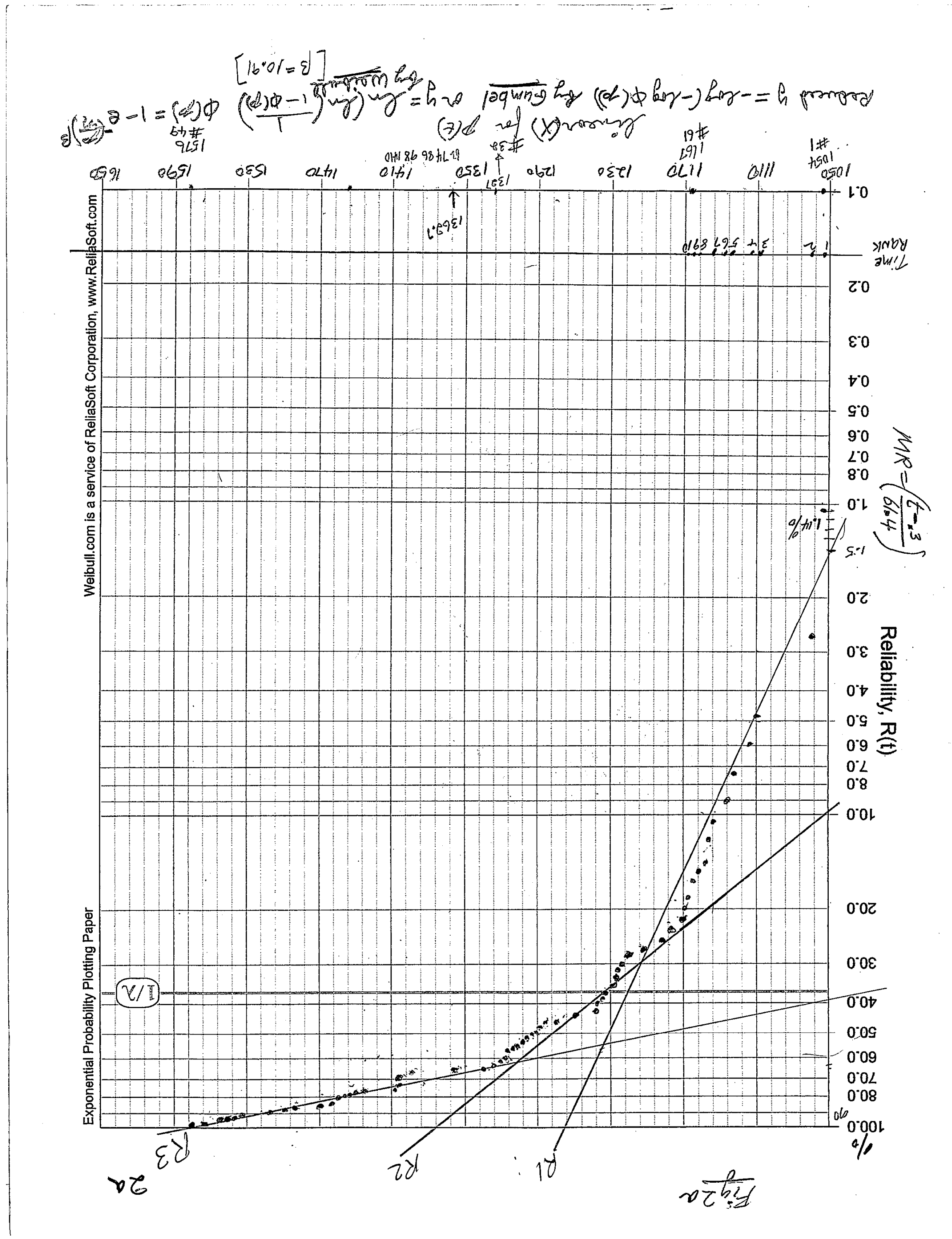
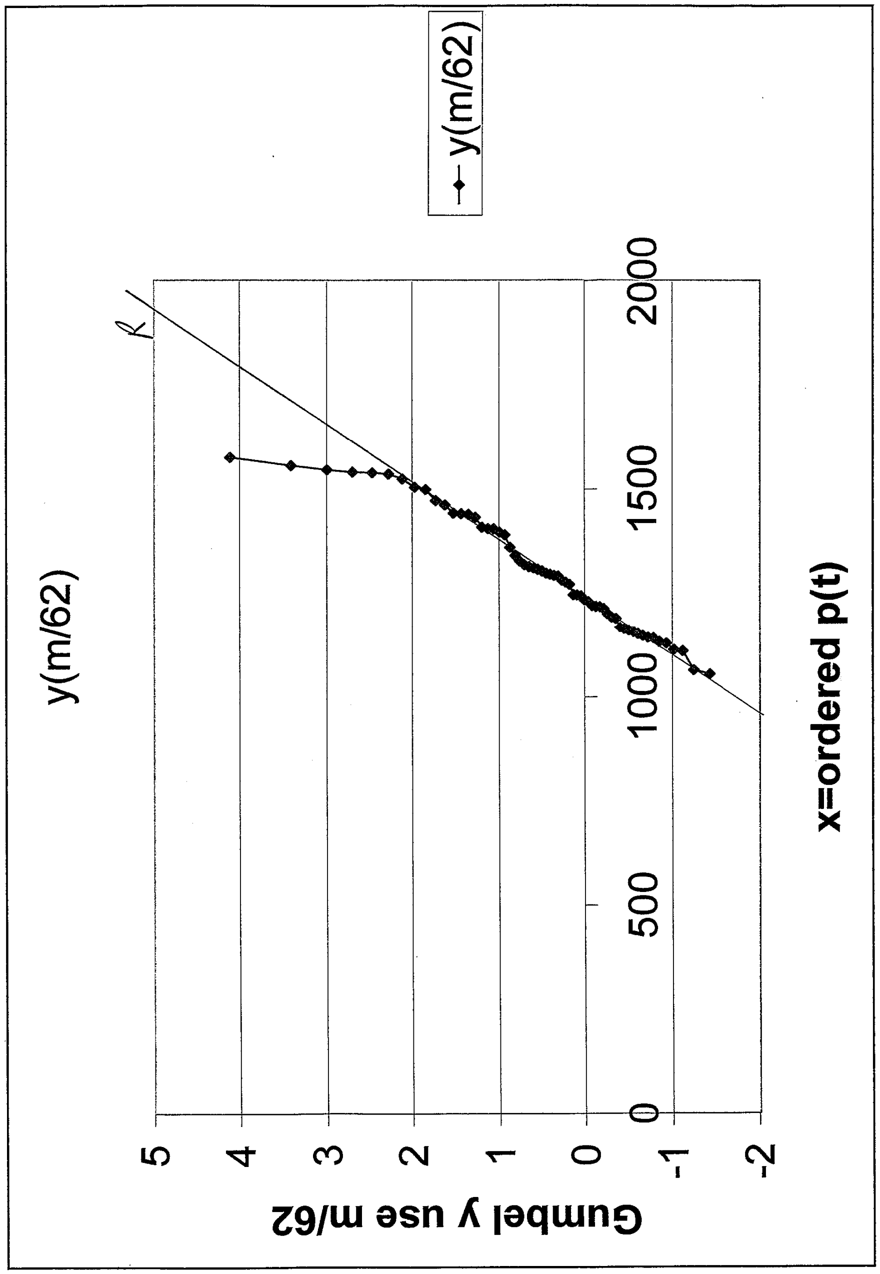


Fig 2a
2a
R1
R2
R3
%
Reliability, R(t)
Exponential Probability Plotting Paper
Weibull.com is a service of ReliaSoft Corporation, www.ReliaSoft.com
1/λ
1.14%
MR = (t − .3)/(61.4)
Time
RANK
1 2 3 4 5 6 7 8 9 10
1368.7
1050 1110 1170 1230 1290 1350 1410 1470 1530 1590 1650
1054 #1
1167 #61
1327 #32
1576 #49
linear(X) for p(t)
Reduced y = −log(−log Φ(p)) by Gumbel or y = ln(ln(1/(1 − Φ(p)))) by Weibull
[β = 10.91]
Φ(p) = 1 − e^(−(…)^β)

Fig 2b


y(m/62)
Gumbel y use m/62
x=ordered p(t)
y(m/62)
R
5
4
3
2
1
0
-1
-2
0
500
1000
1500
2000

24

3a

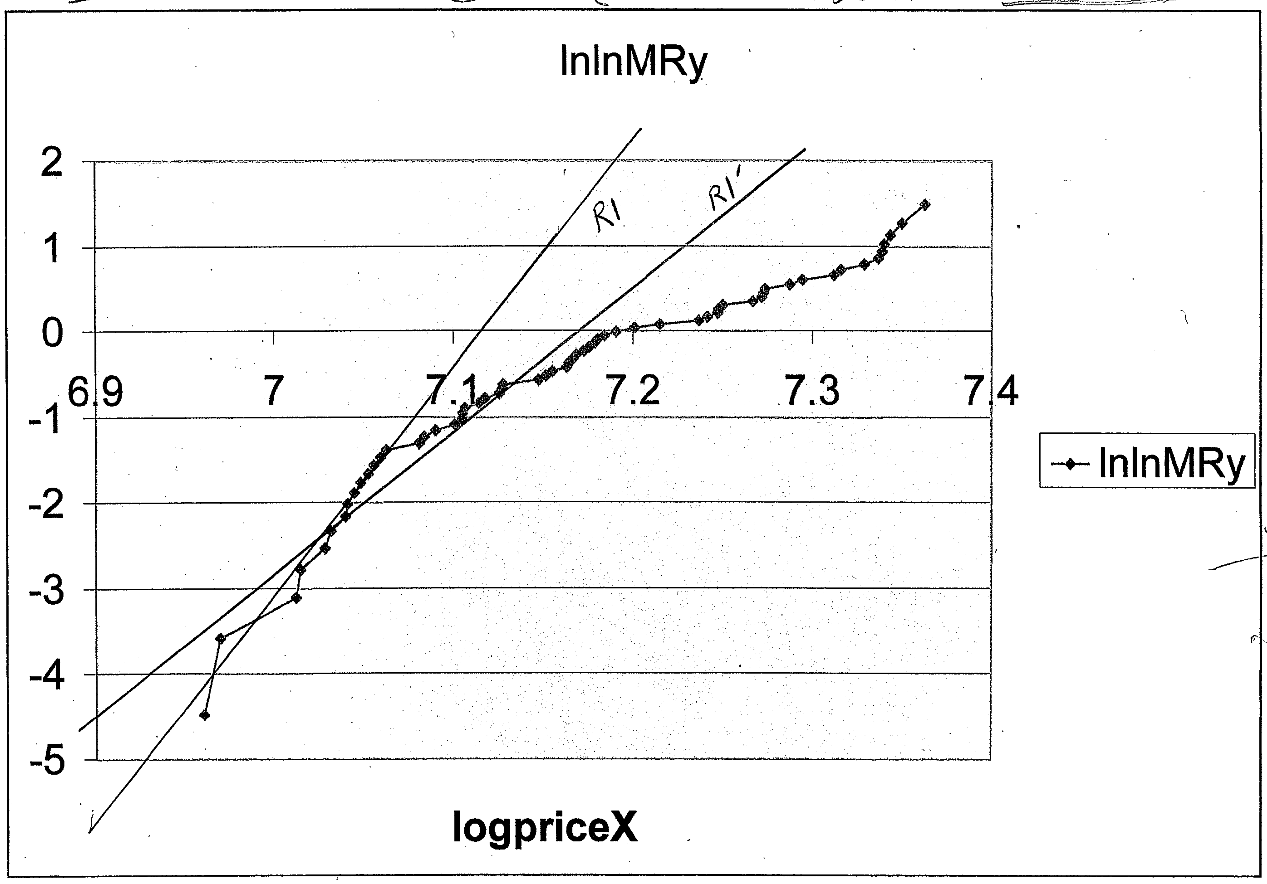
$\lambda = \frac{1}{\eta} = .0323$ may 55

$= .0323 \leftarrow$ 1 to 62 mid 31

(XLS) lnln (MR) MR est $\left(\frac{t-.3}{N+.4}\right)$

Fig 3a 11/18/08 level

Decimals



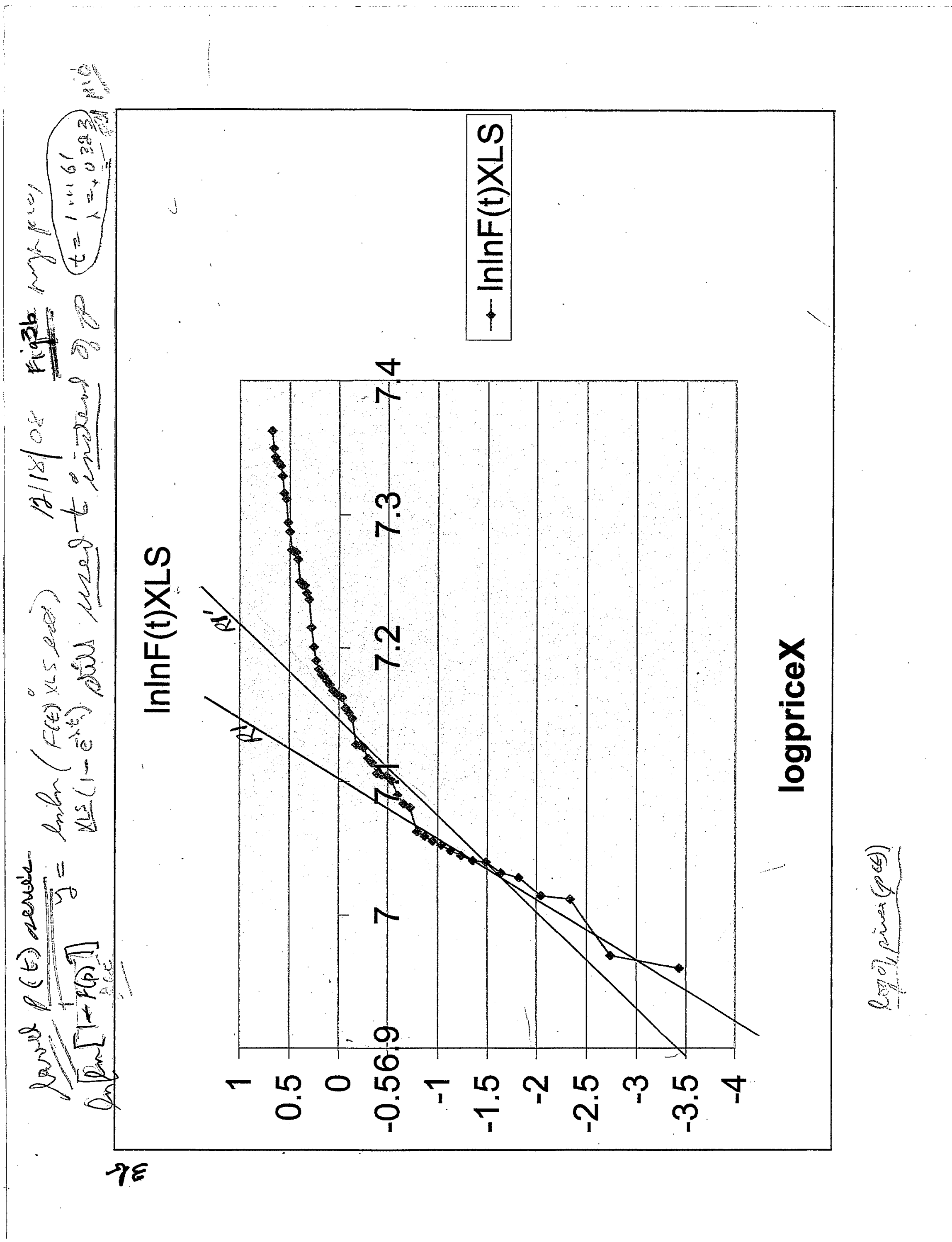
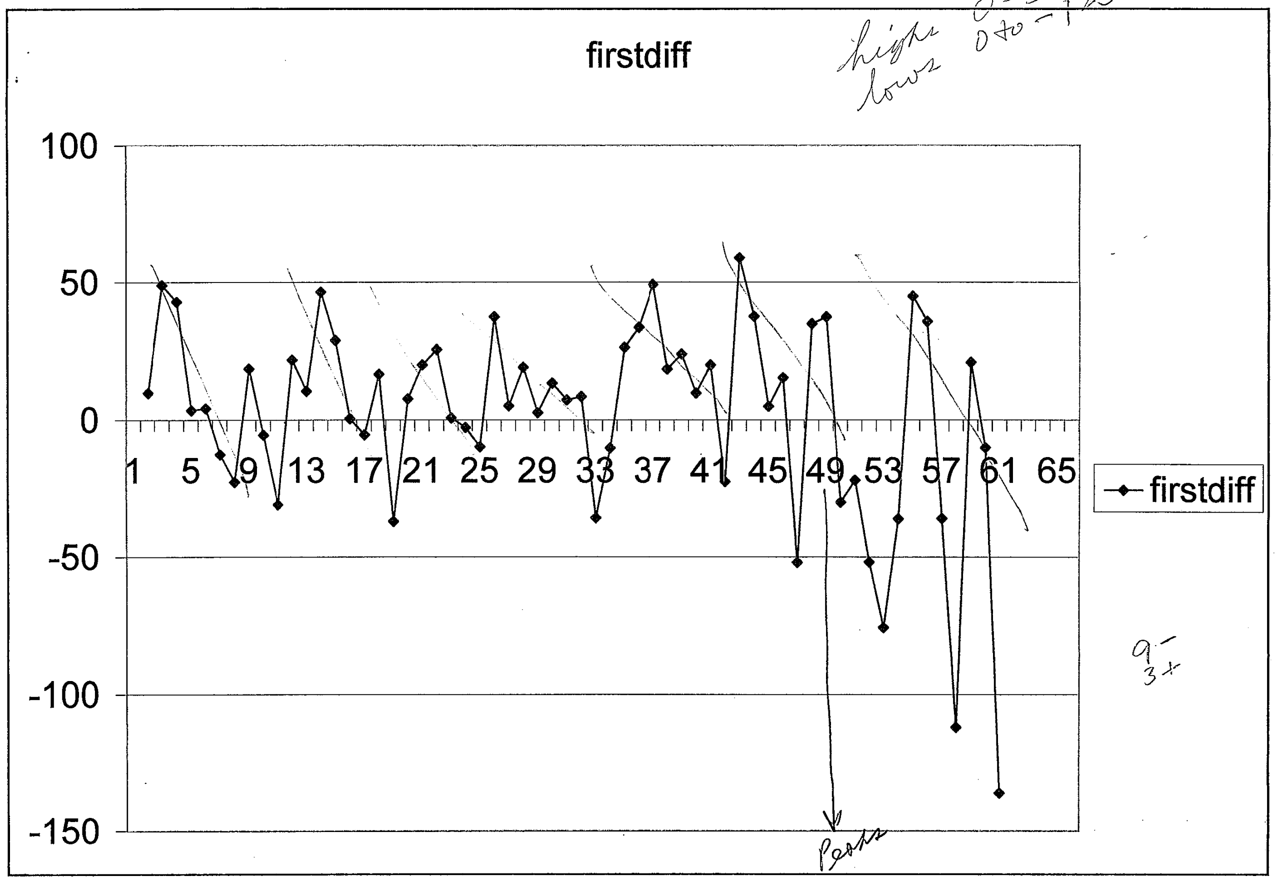

lnlnF(t)XLS
lnlnF(t)XLS
logpriceX
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
6.9
7
7.1
7.2
7.3
7.4
Fig3b
12/18/08

Fig 4

N1) St Pr[illegible] June 03 08.2008 (lag diff) $\Delta P_t = (P_t - P_{t-1})$ natural order of P(t) Fig

4



37 > 0
2 = 0
21 < 0

a few neg's then several pos responses —
cycle repeats

on time, why?

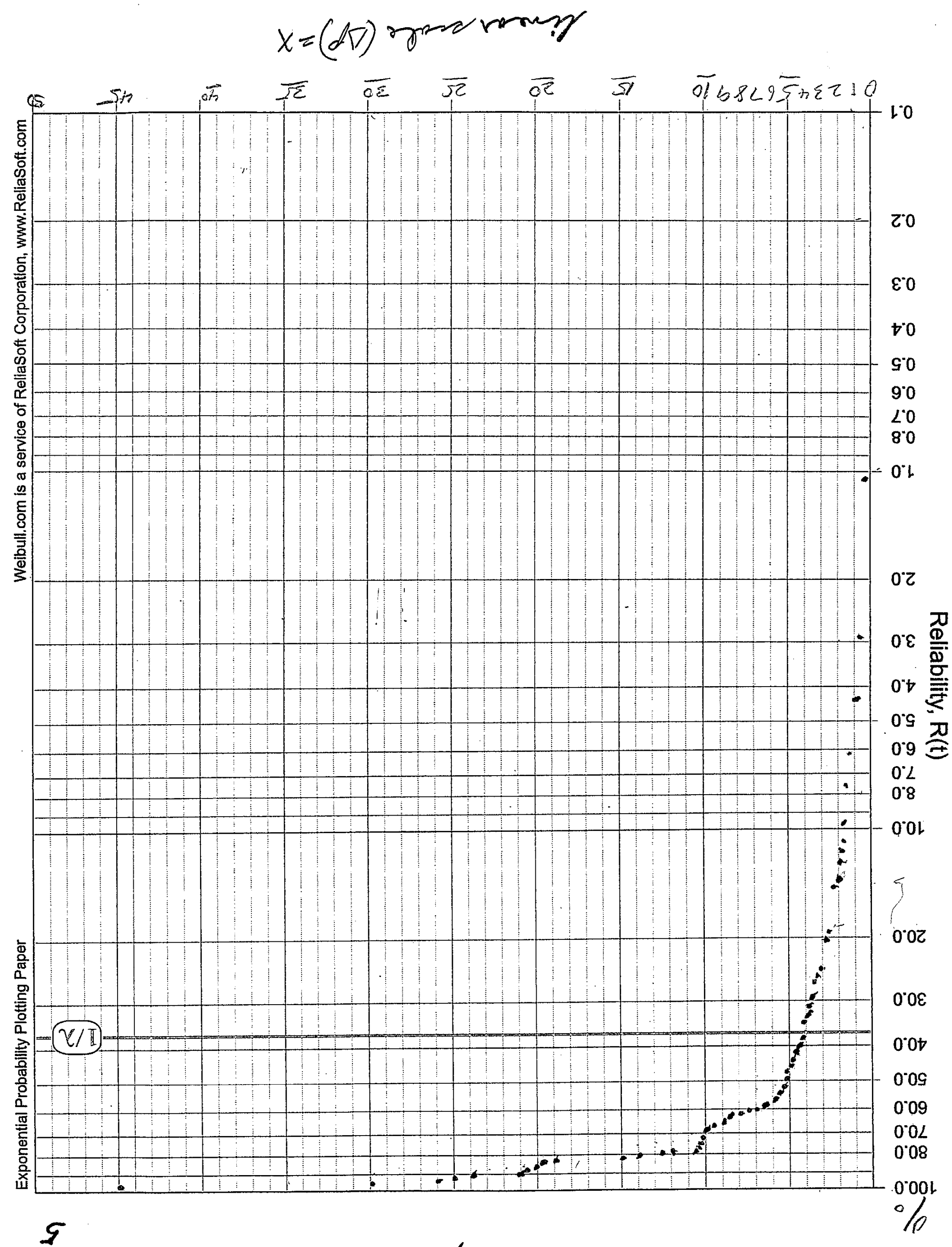
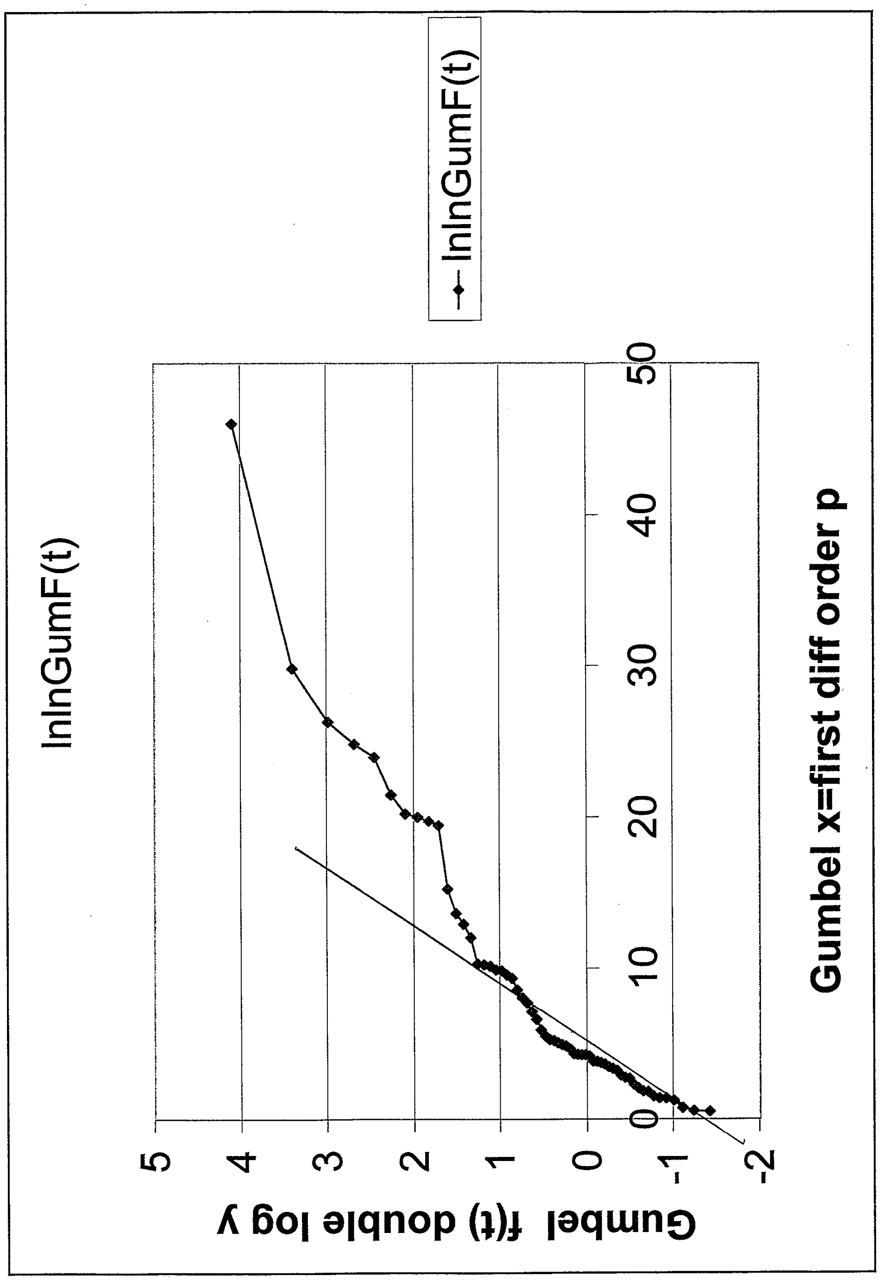

linear scale (ΔP) = X
Exponential Probability Plotting Paper
Weibull.com is a service of ReliaSoft Corporation, www.ReliaSoft.com
Reliability, R(t)
1/λ
%
5

Fig 5

Fig 6a

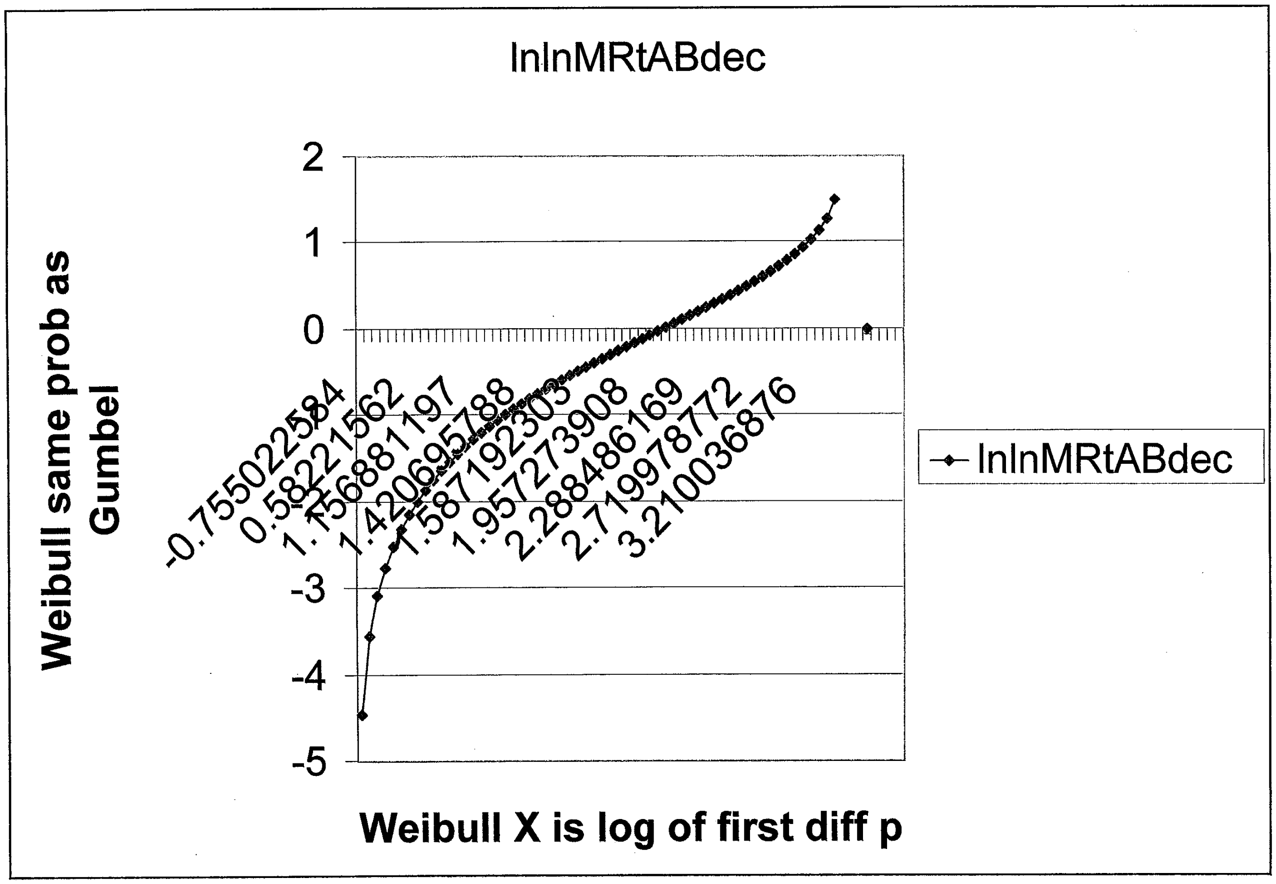

lnlnGumF(t)
Gumbel f(t) double log y
Gumbel x=first diff order p
lnlnGumF(t)
5
4
3
2
1
0
-1
-2
0
10
20
30
40
50

6a

1/8/09 Fig 6b, Weibull, same Y but X = log Δp(t) 1st Diff

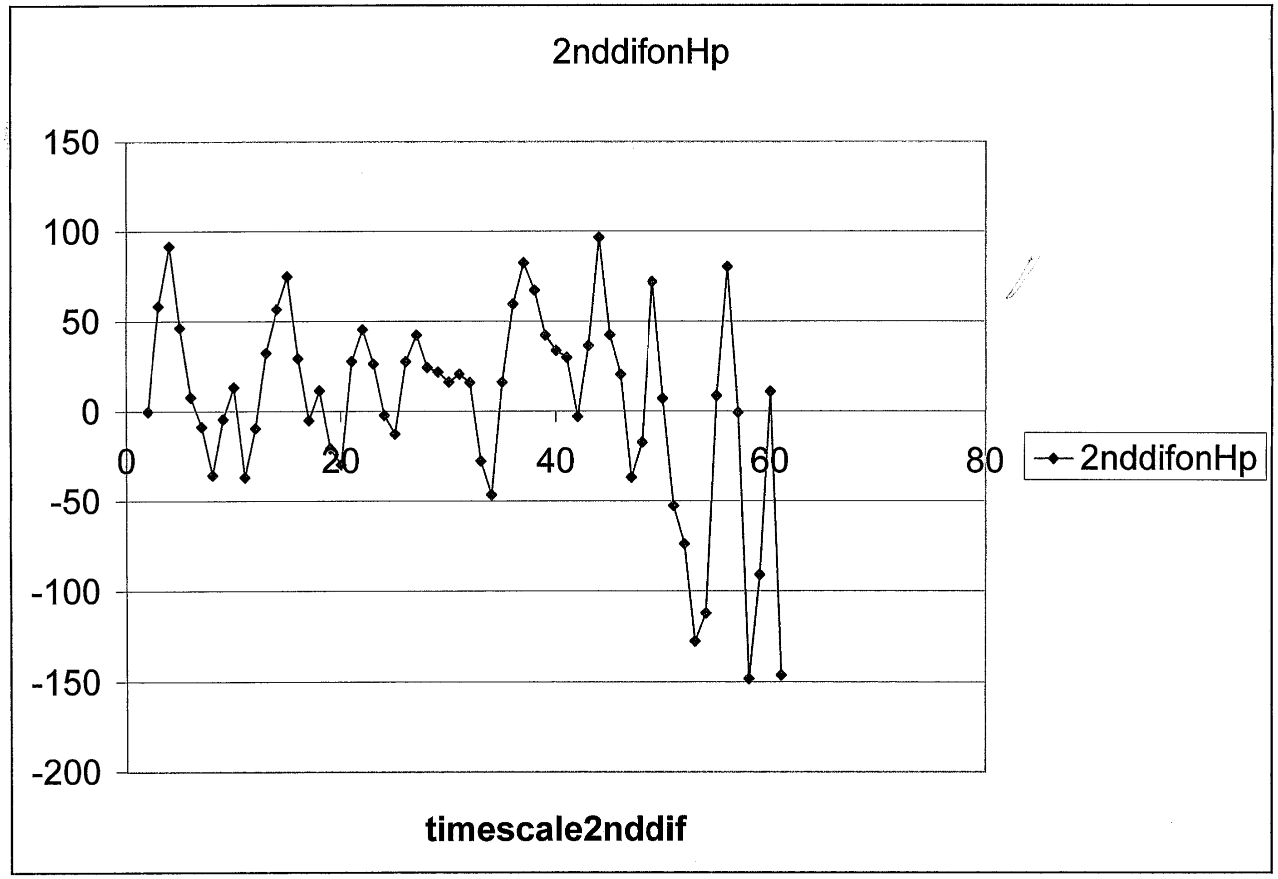
log first diff gold (AA)
6b

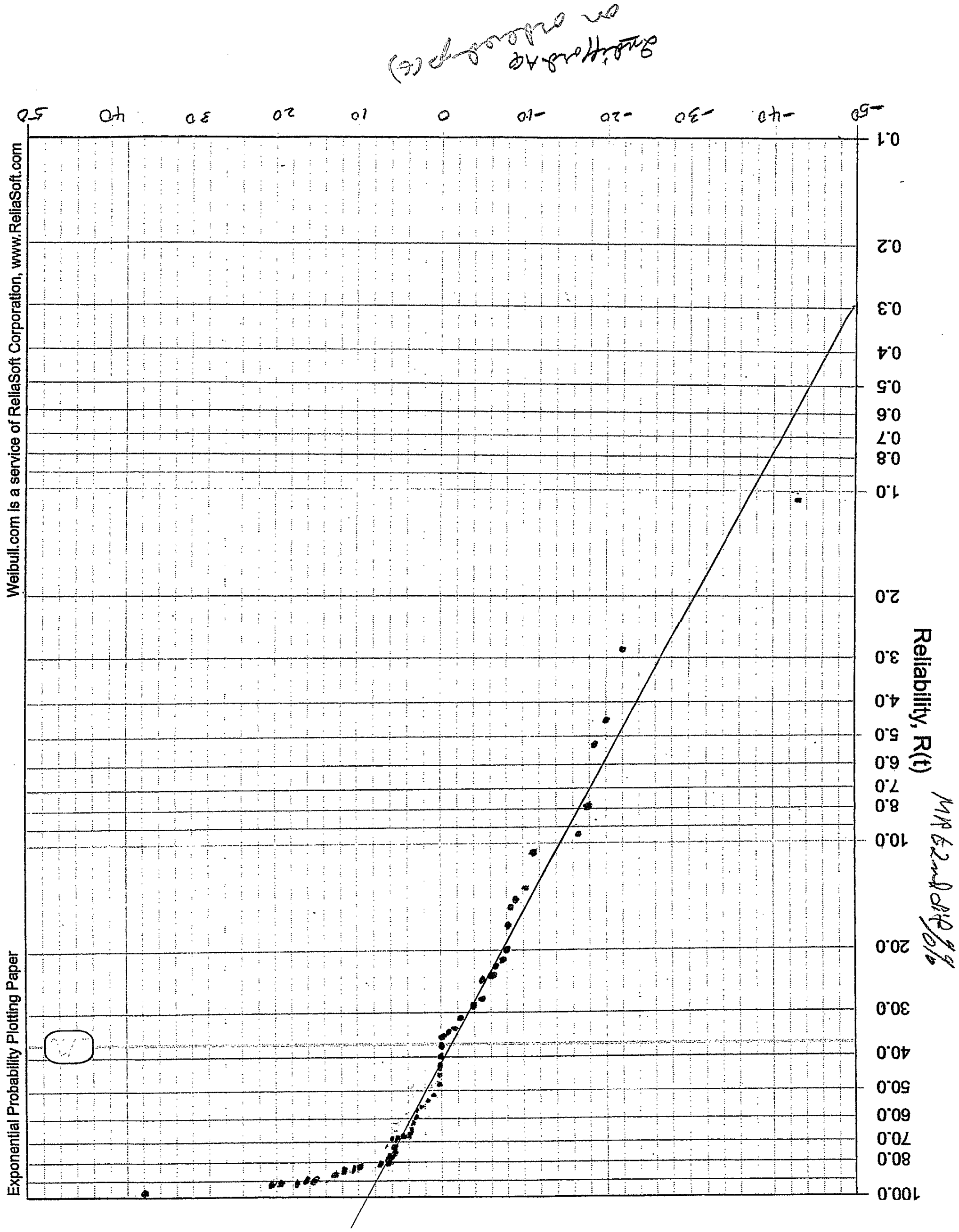
6b



$\Delta 2 = (P_t - P_{t-2})$ $(n = 59)$ Fig 7, natural ordering P(t)

7





Exponential Probability Plotting Paper
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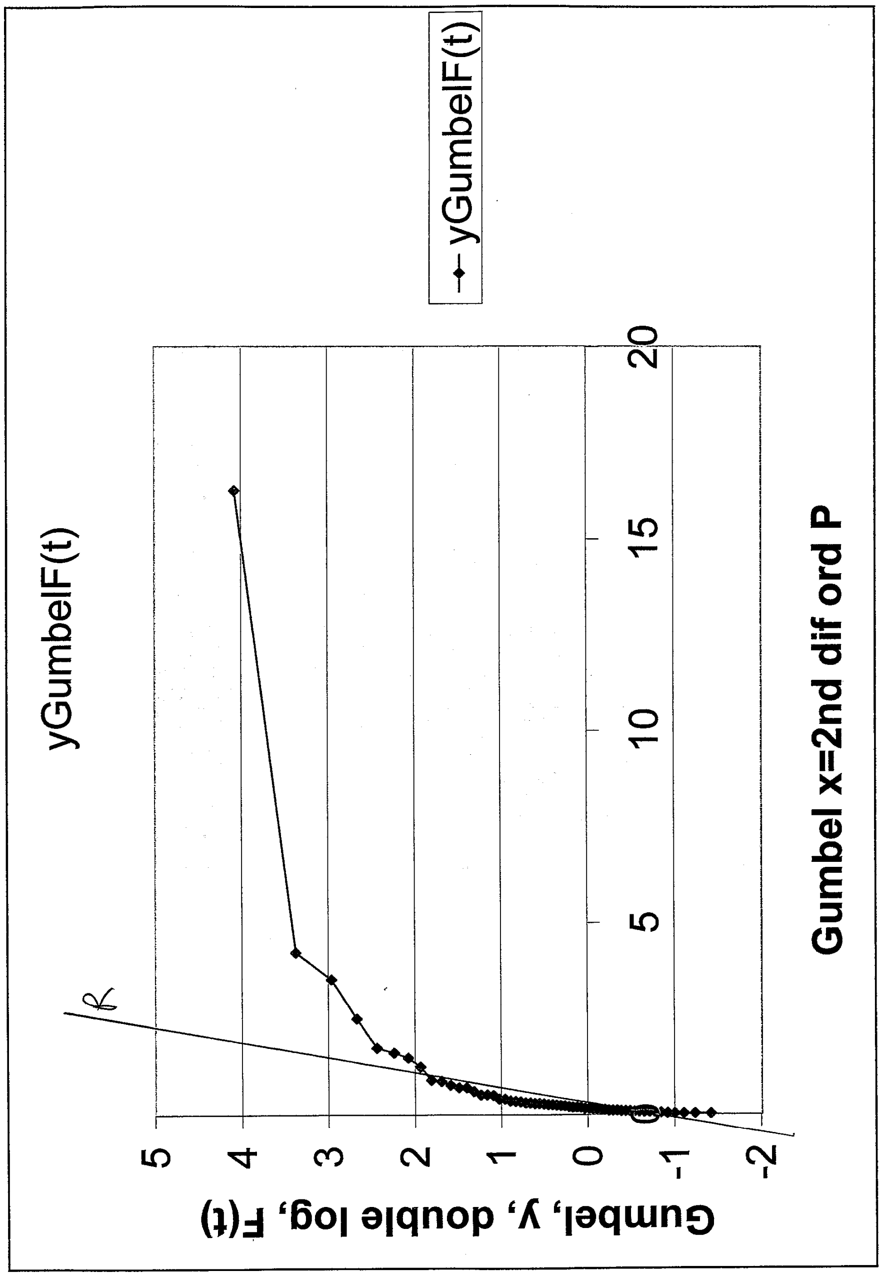
Reliability, R(t)

8

Fig 8

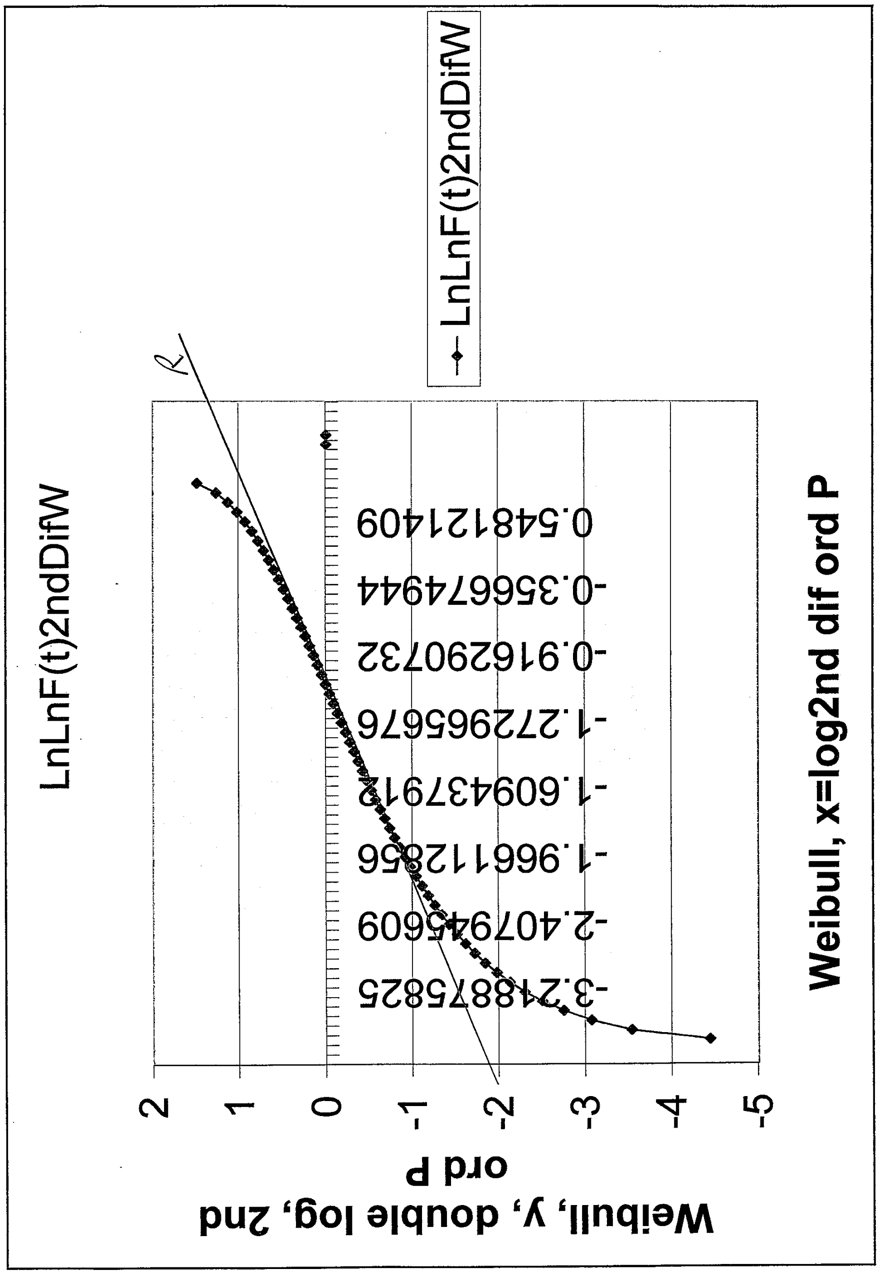
Fig 9a

1/19/09


yGumbelF(t)
yGumbelF(t)
Gumbel, y, double log, F(t)
Gumbel x=2nd dif ord P
5
4
3
2
1
0
-1
-2
5
10
15
20
R

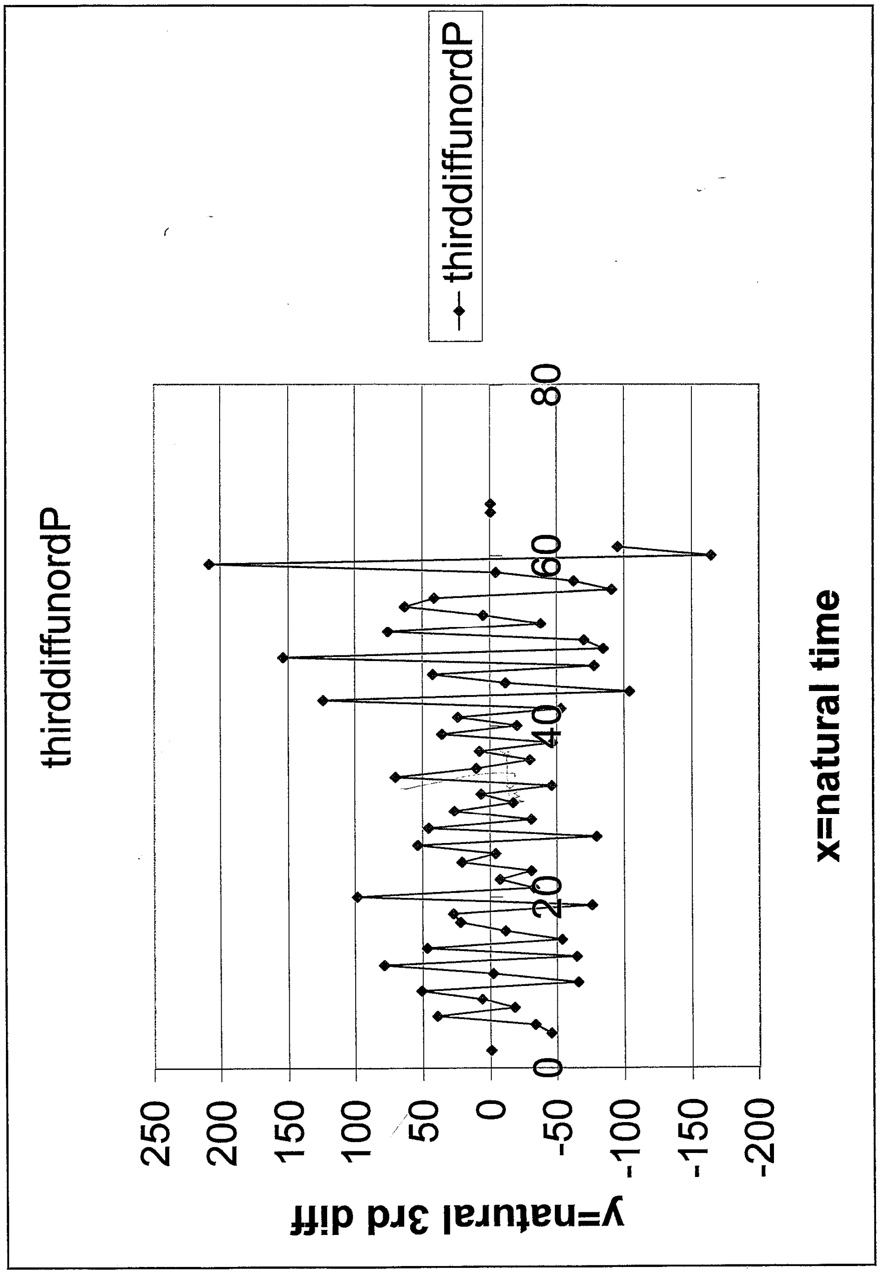
1/19/09

Fig 98



96

fig 10


thirddiffunordP
y=natural 3rd diff
250
200
150
100
50
0
-50
-100
-150
-200
0
20
40
60
80
x=natural time
thirddiffunordP

a)

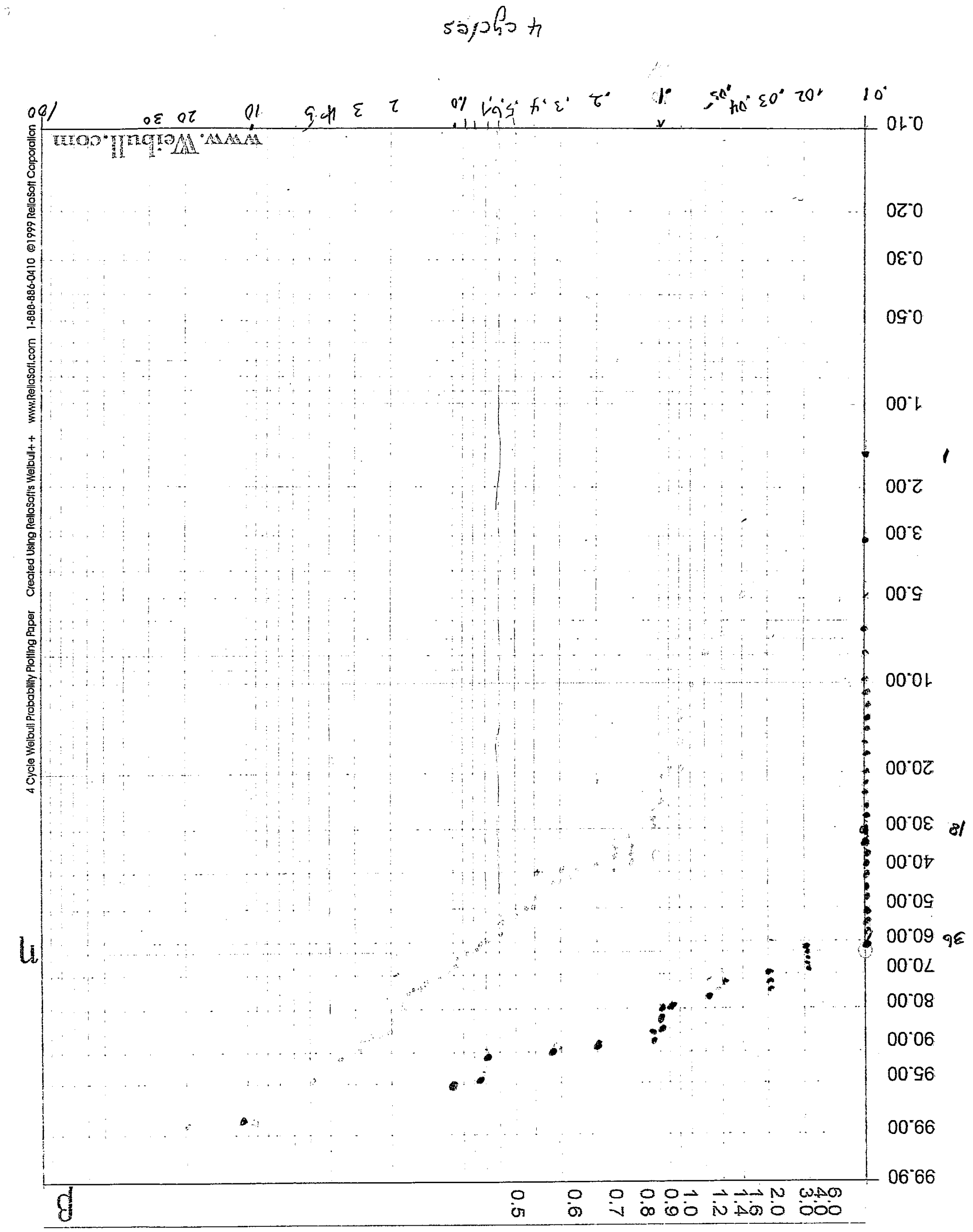
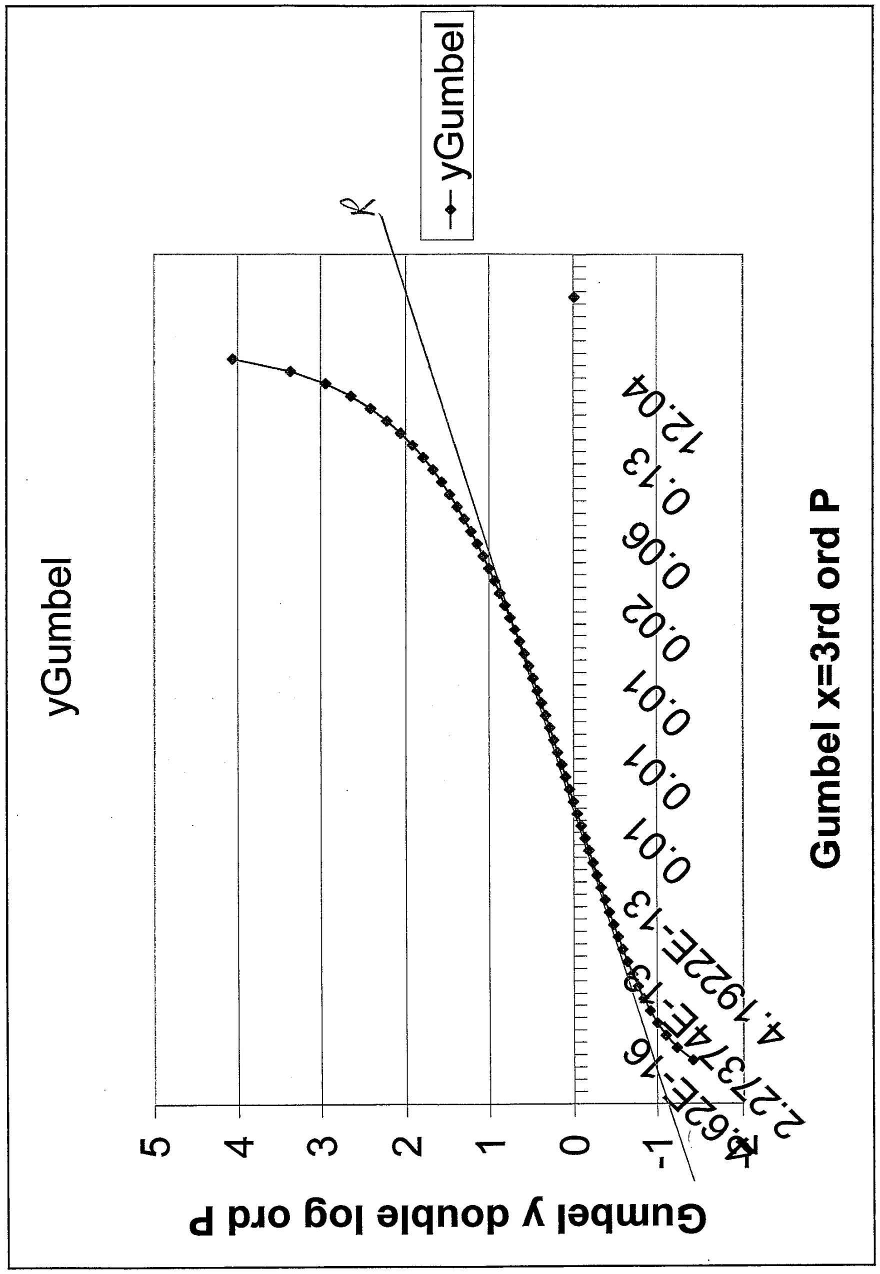

4 cycles
4 Cycle Weibull Probability Plotting Paper Created Using ReliaSoft's Weibull++ www.ReliaSoft.com 1-888-886-0410 ©1999 ReliaSoft Corporation
www.Weibull.com
0.10
0.20
0.30
0.50
1.00
2.00
3.00
5.00
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
95.00
99.00
99.90
0.5 0.6 0.7 0.8 0.9 1.0 1.2 1.4 1.6 2.0 3.0 4.0 6.0
β
η

Fig 11

1/19/09

Fig 12a



12a