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The Power of Reasoning: Experimental Evidence

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Abstract

This paper presents an experimental investigation of how a systematic variation in the cognitive demands on subjects affects the optimal play. The innovation of this paper is the choice of a game, which we call the *Game of Position*. This is a two-player zero-sum game characterized by a dominant-strategy solution that involves iterative steps of reasoning. The equilibrium play is independent of mutual beliefs of players; hence inability of a subject to play the dominant-strategy unambiguously implies the failure of human reasoning prowess. We alter the two parameters of the game to vary the cognitive constraints, as represented by these steps of reasoning, on players. Our main substantive conclusion is that the frequency of the dominant-strategy play sharply increases as we limit the cognitive demands on players.

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JEL Classification: C72, D83, C91.

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1. Introduction

Cognition plays an important role in guiding human behavior. Since the work of Herbert Simon (1955), economists have recognized that individuals are characterized by limited cognitive capacity and therefore are capable of employing finite depths of reasoning. Given this critical element of human decision-making, how limited cognitive capacity interacts with economic decisions and influences quality of economic outcomes is a question of immense importance to economists. This paper experimentally investigates how incidence of the optimal play varies by systematically altering the cognitive demands on subjects in a game of economic interest, which we call the Game of Position, the structure of which is favorable for analyzing subjects' depth of reasoning, as revealed by their choices[1].

The Game of Position is a finite two-player two-outcome strictly competitive game with perfect information. The game has a fixed number of positions that determine the length of the game tree (l). The first-mover (chosen randomly) starts from the first position and players alternate in making decisions. When a player must move, she has the option to add to her current position any number of positions between 1 and n ($n < l$). The game continues with the players alternating in moves, and the player who reaches the last position first wins a fixed prize and the other player receives nothing. Since the game is characterized by positions, and as we will see below that certain positions can be winning positions, we label it the Game of Position[2]. The game is characterized by a first-mover advantage and has a dominant-strategy. Ewerhart (2000) formally shows that any finite

[1] See Crawford (2008), Crawford & Iriberri (2007a), Costa-Gomes & Crawford (2006), Johnson *et al*. (2002), Costa-Gomes *et al*. (2001), Nagel (1998, 1995) for examinations of degree of sophistication of players in various experimental games. For a survey of related research, see Camerer (2003, section 5.6).

[2] We name this class of games so by taking a cue from Baron (1974), who used the term 'position' to describe the solution process of a similar game.

two-player two-outcome strictly competitive game with perfect information must have a dominant strategy. The Game of Position belongs to this class of games.

To illustrate how the game is played, let us consider an example, $g(l = 23, n = 3)$. Figure 1 presents the game. If the first-mover chooses the positions 3, 7, 11, 15, 19 and finally 23, and no other positions in between, then this sequence of choices corresponds to the dominant-strategy play and ensures a sure win for her. The other player's choices thus become entirely irrelevant. To see that the second-mover's choices are irrelevant to a first-mover playing the dominant-strategy, consider the following. Suppose in the above example a first-mover reached 19, and now it is the second-player's turn to move on from there. Then the second-mover can move onto any of these three positions: 20, 21 or 22. Given that the second-mover can choose one of those three positions, the first-mover on the next move can surely secure the final position 23 and thus win the game. So, being at position 19 ensures a victory for a first-mover irrespective of the second-mover's decision. Applying this logic to other winning positions that an equilibrium first-mover must occupy, a second-mover's choices at those positions are of no consequence. If both players know the winning strategy then it is obvious that the final outcome of the game is solely determined by which of the two players is assuming the role of the first-mover[3].

The dominant-strategy play in the Game of Position requires a first-mover to compute certain number of *steps of reasoning* (*SOR*) (*e.g.*, five in $g(23,3)$), working backward from the very last position[4]. For example, a first-mover must realize that to

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[3] A second-mover may also play optimally if a first-mover fails to play according to the equilibrium strategy in the previous move. In that case, a second-mover who has already reasoned out the dominant-strategy play would secure all the winning positions from that point onwards.

[4] Securing all the equilibrium positions in the Game of Position may therefore imply that a player *must* have been able to figure out all the *SOR*. However, there may be an important caveat. A player may not understand the dominant-strategy play at all but may occupy all the wining positions by mere chance.

ensure a win (*i.e.*, to be at 23) she must choose the second last position (*i.e.*, 19). She also must realize that to secure the second last position, she must be at the third last position (*i.e.*, 15) and so on[5]. How many *SOR* an equilibrium first-mover must compute in a Game of Position depends on the two parameters of the game, l and n. For a given l, decreasing (increasing) the n increases (reduces) the *SOR* involved with the dominant-strategy play. Similarly, for a given n, increasing (decreasing) the l increases (decreases) the *SOR* involved with the dominant-strategy play.

Discovering the dominant-strategy play in this game however involves non-trivial mental computations. More importantly, the cognition 'load' is expected to increase, if the number of *SOR* associated with the dominant-strategy play is increased in a controlled manner. This seems natural to expect in view of the widely known fact that increasingly difficult mental computations require progressively more cognitive efforts (Simon, 1957) [6]. Conditional on this maintained assumption, our objective in this paper is to consider a set of Games of Position that systematically vary in terms of the cognitive constraint (defined by the *SOR* for the dominant-strategy play) imposed on players, and study experimentally whether and how fast players learn to apply the full reasoning process. We conjecture that players (with plausibly limited reasoning power) are more

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However the probability of such a pattern of play is extremely low in the games we study in this paper. We will discuss this issue in detail in Section *4*.

[5] Although this scheme of thinking is exceedingly reminiscent of the backward induction algorithm, yet it is not backward induction since other player's optimal choices at each subgame are not referenced in this game's optimal solution process, which is central to the concept of the backward induction solution (Zermelo, 1913). In other words, the solution process considers only one player's dominant-strategy choice at each subgame (provided it exists) regardless of the other player's choice and works backwards in this manner.

[6] A voluminous literature by now has demonstrated that beyond a certain level of complication, humans' logical apparatus ceases to function – a sign of bounded rationality. See Gabaix *et al.* (2006), Weibull (2004), Kahneman (2003), Gigerenzer & Selten (2002), Kahnemann (2002), Mullainathan (2002), Weibull & Lars-Göran Mattsson (2002), Camerer (1998), Ho *et al.* (1998), Rubinstein (1997), Stahl & Wilson (1995), Sargent (1993), Stahl & Wilson (1994), Gottinger (1982), Simon (1982) to get a comprehensive idea of the theoretical and experimental literatures.

likely to play the dominant-strategy play the shorter the l is (holding n fixed) or the larger the n is (holding l fixed). Although it can be argued, for example, that it is very natural that the incidence of the dominant-strategy play would go down as l increases (holding n fixed). This is because a player has to make relatively higher number of decisions in a game with higher l, therefore the scope of making errors would tend to go up. Since an error in this game is directly linked to the cognitive burden involved in computing successive steps of reasoning and the number of such steps strictly increases in l, it further strengthens our argument.

The innovation of this paper lies in our choice of the game in so far measuring individuals' depth of reasoning in the laboratory is concerned. The Game of Position is impervious to potential confounds that may cast any doubt on our measure of an individual's depth of reasoning. Since the game has a solution in dominant strategies, a player's belief about others is completely irrelevant. Failure to play according to the prescription of the game theory unequivocally implies a subject's inability to figure out the dominant-strategy play. Furthermore, given that the optimal play in this game involves discovering certain number of *SOR* and this reasoning process is sequential in nature, it allows us to directly observe how many *SOR* an individual subject is capable of computing in a game. In this sense, our measurement of the individual reasoning process in this game is free from any possible confounds and offers a pure means of capturing the extent of the human reasoning prowess.

One can think of myriad of examples that capture the fact that a decision-maker must decide at each stage what action to take next in order to optimize payoff attained at the end of the decision sequence. Examples include financial planning for retirement, working towards a degree, working for a targeted weight reduction etc. Each of these tasks requires multiple-stage decisions that consist of a series of interdependent stages leading towards a final outcome and cognitive capacity underlies each example. Given the obvious analogy between the optimal solution in the Game of Position and multi-stage decision-making process in the above examples, our experiment aims to shed light on whether individuals can adopt efficient planning method, and how the capacity to carry out a payoff-maximizing planning may interact with the cognitive demand of the task in question. Hence the game offers an ideal environment in which we can test the obvious connection between human reasoning-skill and the quality of economic outcome.

We study four Games of Position that are uniquely defined by the two parameters of the game, l and n (these games are described in Section *3*). The two parameters in these games are varied in a manner in our experiment that allows us to determine the effect on the dominant-strategy play of changing l while holding n fixed and vice-versa. Furthermore, the four games can be ranked in ascending order of the number of *SOR* associated with the dominant-strategy play.

The results from the experiment provide substantive evidence that the frequency of the dominant-strategy play sharply increases as we limit the cognitive demand on players. The data confirm that the incidence of the equilibrium play is inversely related to the decision-tree length (holding n constant), but decreasing the maximum number of positions that a player on-move can add (holding l fixed) does not have any unequivocal effect on the dominant-strategy play. The analysis of the equilibrium play data in general implies that optimal dynamic planning can be significantly impeded by the complexity of the task related reasoning process. Learning dynamics indicate that initially many subjects won a game by computing only the last few steps of the sequential reasoning process, but after gaining enough experience majority of them learnt to decipher additional steps of reasoning and played the dominant-strategy. We conjecture that this pattern of play may arise due to subjects' inability to carry out all the steps of reasoning except the last few steps in initial stages of the play, or may be due to subjects' initial tendency to play according to an ad-hoc forward-looking approach. Thus learning the optimal strategy by gradual experimentation is another way to think of our results. The rest of the paper is organized as follows. Section *2* discusses the relevant literature. Section *3* lays out the experimental design, Section *4* presents the results, and Section *5* concludes.

2. Relevant Literature

This paper is closest to the experimental Beauty Contest game (BCG) literature (Nagel, 1995). In a typical BCG, a group of players simultaneously pick a number between [0, 100]. Whoever is closest to 2/3 of the average number wins a fixed sum of money. In case of a tie, winning prize is split among players. Iterated elimination of weakly dominated strategies predicts that each player should choose 0.

However, results from Nagel (1995) are at odds with the equilibrium prescription. The average number chosen was around 35, and a handful of subjects chose 0. The experimental data from this game is often used as evidence of an *individual*'s limited reasoning ability. However such interpretation may be flawed. For example, a player endowed with deep reasoning power may still choose a number greater than 0 if she holds non-equilibrium beliefs about others' bounded ability to reason out[7]. Since discovering the optimal individual choice in this game needs consideration of players' mutual beliefs about each others' rationality, laboratory results of this game may not necessarily reflect an individual's lack of ability to carry out certain rounds of iterated reasoning and thereby lead to an erroneous conclusion about subjects' depth of reasoning[8]. The Game of Position avoids such confounds. Failure to play as per the dominant-strategy play unambiguously signifies a player's incapacity to apply the full chain of reasoning process.

There are very few papers that demonstrate that planning horizon may adversely affect the equilibrium play in other games. Aymard & Serra (2001) find in a resource extraction experiment that the length of a game may significantly influence subjects' understanding of the optimal strategy. Experiments by Johnson and Busemeyer (2002) suggest that subjects, involved in a multiple-stage decision-making game, regularly violate dynamic consistency principle at a higher rate as the length of the decision tree increases. Blume & Gneezy (2000) on the other hand show that in a coordination game that lacks an *a priori* common-knowledge description, optimal learning can be severely impeded by the complexity of the coordination task.

[7] See Camerer (2003, *p*.17) for an illustrative account of this non-equilibrium belief. Grosskopf & Nagel (2007) attempt to experimentally distinguish between two possible reasons that may explain why subjects do not play the equilibrium prediction in a two-person BCG.

[8] There are two components to one's choice in a BCG. First, there is one's rationality and second is one's beliefs about other players' rationality. Grosskopf & Nagel (2008) find that even in a two-player BCG in which one step of reasoning leads to a weakly dominant choice of 0, overwhelming majority chose dominated strategies. This is surprising because this game is strategically equivalent to a game of symmetric Bertrand price competition.

Gneezy *et al.* (2007) explore games similar to the Game of Position with l = 15 or 17 and n = 3 or 4[9]. Although the games are similar, their research question is entirely different from that of ours. We do not record response times of subjects to understand how quickly they understood the backward induction process as in Gneezy *et al.* They do not study the change in the frequency of the dominant-strategy play in response to changes in the cognitive burdens on players, determined by the parameters of the game, as we do. Mckinney & Van Huyck (2006, 2007) also examine depth of strategic reasoning in Nim games. However a Nim game differs from the Game of Position in a way each sub-game is defined in both the games. Therefore our paper constitutes the first effort that aims to identify the limits of human reasoning in a game that differs from a Nim game.

3. Design

Our experiment has a 2X2 design (see Table 1). Specifically, we consider l = 15 & 23, and n = 3 & 4. Crossing the two criteria, we obtain the following four games: *g(15,4)*, *g(15,3)*, *g(23,4)*, and *g(23,3)*. Each game is therefore distinguished by the two parameters, and the games vary either in l (holding n constant) or in n (holding l constant). Each of the four games has a first-mover advantage and a dominant-strategy solution. The equilibrium positions for the four games are as follows[10]:

***g*(*15, 4*):** 5, 10, and 15.

***g*(*15, 3*):** 3, 7, 11, and 15.

***g*(*23, 4*):** 3, 8, 13, 18, and 23.

[9] We became aware of this paper after conducting our experimental sessions.

[10] If there are x number of equilibrium positions in a game, then the number of *SOR* to play the dominant strategy in that game is equal to (x-1).

***g*(*23, 3*):** 3, 7, 11, 15, 19, and 23.

The number in each cell in Table 1, corresponding to a particular *g(.)*, represents the number of positions (*SOR*) that a first-mover must acquire (compute) if she were to play optimally. For example in *g*(*15,3*), a first-mover needs to acquire *four* winning positions, and thus finish *three SOR* if she were to play optimally. In contrast, in *g*(*23,3*), a first-mover needs to acquire *six* winning positions, and thus compute *five SOR* if she were to play optimally. Thus, increasing the *l* of a game (and keeping the *n* constant) appears to increase burden on cognition substantially. In fact, in this case it increases cognitive burden by two *SOR*. Similarly if one moves from *g*(*15,4*) to *g*(*15,3*)**,** thus decreasing the *n* of a game and keeping the *l* constant, this causes the *SOR* to go up by one[11]. Since we already know from Ho *et al.* (1998) that in a BCG subjects typically use one to three *SOR*, playing optimally in some of our games may turn out to be cognitively burdensome for our subjects.

However subjects may not always play a game as per the equilibrium prescription. Another possible way to play such a game may be to temporarily avoid the complex mental computations needed to search for the optimal strategy and instead adopt a forward-looking approach. That is, a subject in initial stages of play may choose to decide on a particular position by trial and error process, and see the outcome of this initial decision. She can again consider what she is going to do once the other player makes her decision, and so on. This may constitute a natural response from a subject to whom the game appears unknown and complex. We know however that this method of

[11] However reducing the *n* may in fact ease a subject's cognitive burden. For example, in *g*(*15,3*) a subject can move on to at most three positions ahead from her current position. In contrast, in *g*(*15,4*) a subject can move on to at most four positions ahead from her current position, thus adding one more position in her potential choice set, which may prove cognitively burdensome for a subject in deciphering the dominant-strategy.

analysis is bound to be unsuccessful in principle if the other player has already figured out the optimal strategy and it may be less rewarding in practice, yet many subjects might adopt such a strategy. Although after playing a game for several times a subject may be expected to *learn* the dominant strategy - a scope that our experimental design permits.

The experimental sessions were run in the CBEEL at the University of Calgary. The subjects of the experiment were undergraduate students registered at this university. The experiment consisted of 8 sessions, two sessions for each treatment. A session consisted of 16 participants playing a particular *g(.)* for 15 periods. Thus a total of (16 × 2 × 4) 128 subjects participated in the experiment. Each subject participated in only one session. In each period a subject played the same game with a new opponent, *i.e.*, no subject was ever matched with any other subject more than once. This perfect-stranger matching scheme helps in retaining the one-shot character of a game while permits only *game* and *subject pool* specific learning. Furthermore, this matching scheme will allow us to obtain multiple observations on each individual's behavior.

Since subjects in our experiment might need to put in significant cognitive effort to understand the optimal winning strategy and insignificant monetary incentives might induce subjects to put in less effort, we decided to strongly incentivize the paying scheme by paying $3 for each win and $0 otherwise. So in a session under each treatment, which never lasted for more than 30 minutes, a subject could win up to C$45, excluding a show up fee of C$5. Given that the minimum hourly wage in the province of Alberta is approximately C$8, winning up to C$45 in roughly 30 minutes is believed to provide more than adequate incentive for subjects to put in enough effort to search for the equilibrium strategy[12].

The experiment was conducted using the computer. The subjects were not permitted to communicate with each other once the experiment had commenced other than by selecting their move on the computer screen using a mouse. At the beginning of each period, the computer randomly assigned each player one of the two possible roles: the *first-mover* or the *second-mover*. Each game is displayed as a series of numbers in boxes starting from 1 to l, depending on what value l may have taken in that session. When a player clicks on a box or boxes, those many boxes change their color to confirm her choice. This change of color also informs the rival player about the previous move and notifies that it is now her turn to make a decision. Each session began after subjects were given sufficient time to concentrate on the instructions. A session began after an experimenter answered all questions regarding the experiment. We did not impose any time limit on subjects during play, thus ensuring that time does not play any conceivable role in influencing subjects' decisions in any treatment.

4. Results

The results from our experiment are discussed in the following two subsections. In subsection *4.1*, we test our main research question using the experimental data from the four treatments. In subsection *4.2*, we focus on whether and how subjects learn to carry out increasing number of *SOR* as play progresses in each treatment.

4.1 Equilibrium Play

[12] Ho *et al.* (1998) find that increasing the stake size has the effect of lowering number choices (i.e., increasing the number of *SOR*) in a BCG.

We start our analysis with the following question: In a given treatment how many of the total games were won by the first-movers and how many by the second-movers? Table 2 presents the evidence. In this analysis we do not distinguish whether a specific mover won a game by playing according to the dominant-strategy or not, thus only providing a *rough* estimate of the effects of the two game parameters on the first-mover advantage. In *g(15,4)*, 178 of 240 games (74%) are won by first-movers whereas in *g(23,4)* this number goes down to 148 (62%). An analogous comparison between *g(15,3)* and *g(23,3)* shows a similar effect, but the number decreases slightly from 150 (63%) to 148 (62%). Investigating further the effect of varying the *n*, we find that between the treatments *g(15,4)* and *g(15,3)* the percentage of games won by first-movers goes down from 74 to 63. In contrast, these percentages do not change between the treatments *g(23,3)* and *g(23,4)*. Overall, as the number of *SOR* (associated with the dominant-strategy play) increases across the four treatments, the number of games won by first-movers decreases, and this decline is weakly monotonic in nature.

We now proceed to consider our main research question posed in Section *1*. For this purpose we introduce an index, which we call the *Index of the Power of Reasoning* (*IPOR*). *IPOR* can assume an integer value between 0 and m (both inclusive) for a player. m represents the *SOR* that a player must compute in a game to play according to the dominant-strategy prediction, and m = 2, 3, 4, and 5 in *g(15,4), g(15,3)*, *g(23,4)*, and *g(23,3)*, respectively. For example, a player with an *IPOR* = 0 implies that she won a game without carrying out a single step of reasoning. A player with an intermediate value of *IPOR* (*i.e.*, 0 < *IPOR* < m) implies that she won a game by carrying out only some of the *SOR* (from the end of the decision tree) involved in the dominant-strategy play in that game. As a result of this classification, a first-mover with an *IPOR* = *m* in a given game means that the first-mover is a dominant-strategy player[13]. In other words, that first-mover had fully exploited the advantage of moving first in that game.

Having fixed these definitions, we can now focus on the following question: Does altering the *l* of a Game of Position while keeping the *n* constant influence the occurrence of the dominant-strategy play? Table 3 reports the aggregate equilibrium statistics. Out of a total of 240 games in *g(15,4)*, 156 (65%) are played according to the dominant-strategy (by the first-movers), while in *g(23,4)*, only 72 of 240 (30%) games are played according to the dominant-strategy. So, on an aggregate basis a two-step increase in the reasoning process between these two games, from two to four, lowers the incidence of the dominant-strategy play by 35 percent. Focusing on the other two treatments, the corresponding figures are 128 (53%) and 92 (38%) in *g(15,3)* and *g(23,3)*, respectively. Again a two-step increase in the reasoning process, from three to five, decreases the percentage of the dominant-strategy play by 15 percent. Thus a two-step increase in the iterated reasoning process in the two cases leads to significantly different levels of reduction in the dominant-strategy play[14]. Therefore, *our first main result is that an*

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[13] Note that a first-mover may occupy all the equilibrium positions in a game but she still may not play according to the dominant-strategy. Thus she would not qualify as an equilibrium player in our analysis. To see this let us consider an example. Suppose in *g(15,4)* a first-mover starts at position 1 and moves onto position 3, a move not in accordance with the dominant-strategy play if that player had understood the optimal strategy. The second-mover however moves from position 3 to position 4, indicating that she did not realize the dominant-strategy play either. Now our first-mover moves on to positions 5, 10, and 15 in her next three decisions, given the second-mover's decisions in between. In this case the first-mover wins the game but does not play according to the dominant strategy since she did not move from position 1 to position 5 in just *one* step. In the following analysis we regard a first-mover as an equilibrium one (*i.e.*, *IPOR* = *m* for that player in a given game) if she occupies *only* the equilibrium positions in a game and *does not* occupy any other position(s) between any two consecutive equilibrium positions in that game. This criterion applies to second-movers as well.

[14] The difference in the proportion of first-movers playing flawlessly in *g(15,4)* and the proportion of first-movers playing flawlessly in *g(23,4)* is highly significant (Z statistic = 7.59, $p = 0$). The difference in the proportion of first-movers playing flawlessly in *g(15,3)* and the proportion of first-movers playing flawlessly in *g(23,3)* is also highly significant (Z statistic = 3.21, $p = 0.00$).

increase in the l of a Game of Position, while holding the n fixed, leads to a statistically significant reduction in the incidence of the dominant-strategy play at an aggregate level.

Next we turn to the question whether altering the *n* of a Game of Position while keeping the *l* constant affects the occurrence of the dominant-strategy play? To answer this question, we refer back to Table 3. The aggregate data indicate that the proportion of the dominant-strategy play drops from 65 to 53 percent between the treatments *g(15,4)* and *g(15,3)*. However the same figure registers a 8 percent (from 30 to 38 percent) increase between the treatments *g(23,4)* and *g(23,3)*. So, a one-step increase in the reasoning process, induced by a reduction in *n* while keeping *l* fixed, generates entirely different effects in games with different *l*[15]. Thus *our second main result is that a decrease in the n of a Game of Position, while holding the l fixed, leads to a statistically significant reduction in the incidence of the dominant-strategy play for the treatment pair g(15,4) and g(15,3) at an aggregate level. However a similar change in the n leads to a statistically significant increase in the incidence of the dominant-strategy play for the treatment pair g(23,4) and g(23,3) at an aggregate level.*

Overall this provides compelling evidence that the dominant-strategy play may have been considerably influenced by a change in *l* or *n*[16]. Additionally, ranking the games from the easiest to the most difficult (in terms of the number of *SOR* as per the dominant-strategy play), we find that the occurrence of the dominant-strategy play steadily declines as one moves from *g(15,4)* to *g(15,3)*, and to *g(23,4)*, indicating a

[15] The difference in the proportion of first-movers playing flawlessly in *g(15,4)* and the proportion of first-movers playing flawlessly in *g(15,3)* is highly significant (Z statistic = 2.58, $p = 0.01$). The difference in the proportion of first-movers playing flawlessly in *g(23,3)* and the proportion of first-movers playing flawlessly in *g(23,4)* is also highly significant (Z statistic = 1.75, $p = 0.04$).

[16] The probability that a player has *not* understood the dominant-strategy play at all but did occupy all the wining positions is $(1/4)^3$ in *g*(*15,4*). The same probabilities in the games *g*(*15,3*), *g*(*23,4*), and *g*(*23,3*) are $(1/3)^4$, $(1/4)^5$, and $(1/3)^6$, respectively. Given that these probabilities are extremely low, these aggregate statistics provide some enough reason to believe that subjects might have understood the optimal play.

negative relationship between the frequency of the dominant-strategy play and the number of *SOR* (a possible indicator of cognitive burden). However thereafter it registers a slight increase in *g(23,3)*, thus making the relationship non-monotonic.

The negative difference in the proportion of the dominant-strategy play between *g(23,4)* and *g(23,3)* is somewhat counterintuitive because *g(23,3)* involves an extra step of reasoning relative to *g(23,4)*, and therefore requires presumably more mental computations. However the incidence of the dominant-strategy play is statistically higher in *g(23,3)* than in *g(23,4)*. So, *g(23,4)* turns out to be *ex post* the most difficult game for our subjects to play. Why? We offer the following conjecture. Let us continue with our maintained assumption that the dominant-strategy play in games with higher l are cognitively more challenging than that of in games with lower l. Now *g(23,4)* provides a subject with an option to move on to one of the four positions ahead from her current position, whereas *g(23,3)* provides a subject with an option to move on to one of the three positions ahead from her current position. Given that the games characterized by longer decision tree are already more difficult to play, an increase in n from 3 to 4 in games with $l = 23$ may translate into higher marginal cognition load for a subject in terms of how many potential choices she has in the game with $n = 4$. This may in turn adversely affect the process of deciphering the dominant-strategy play in *g(23,4)* relative to *g(23,3)*. In contrast, an increase in n from 3 to 4 in games with $l = 15$ may not necessarily translate into higher marginal cognition load for a subject, simply because games with lower l are seemingly cognitively less challenging. Moreover, in *g(15,4)* the equilibrium positions take the form of multiple-of-five, which may help a subject even more to decipher the dominant-strategy play as compared to *g(15,3)*.

Recall that a second-mover who has already reasoned out the dominant-strategy play will occupy all the remaining equilibrium positions if a first-mover failed to play according to the dominant-strategy in the previous move. To capture the effect of a change in either l or n (and keeping the other parameter constant) on the equilibrium play by the second-movers, we conducted the following analysis.

In this analysis we classify a second-mover as an equilibrium player if she occupied all the equilibrium positions from the point where she obtained the *earliest ever* opportunity to do so. However when a second-mover wins a game by securing only the final position (*i.e.*, $IPOR = 0$) and that constitutes her earliest available opportunity to secure an equilibrium position in that game and therefore does not compute a single step of reasoning, we are never sure whether that second-mover realized the dominant-strategy play or not[17]. Hence we do not consider a second-mover with $IPOR = 0$ as an equilibrium player. To sum it up, a second-mover is regarded as an equilibrium player in the following analysis only if she did not miss any opportunity to occupy all the wining positions from the earliest available point and computed at least one step of reasoning to reach the final position (*i.e.*, secured at least the last two equilibrium positions in a game; $m \geq IPOR \geq 1$ for that player).

One should be cautious when using the above definition of the equilibrium second-mover, as this notion may be an imperfect measure of a second-mover's reasoning capacity. Apart from the case mentioned in the immediately preceding paragraph, there could be few more potential situations when we cannot know for sure

[17] There are a total of 10 (1%) such cases out of a total of 960 cases in our experiment where a second-mover won a game without carrying out a single step of reasoning and that constituted her first ever opportunity.

whether a second-mover realized the equilibrium strategy or not. For example, a second-player may secure positions 18 and 23 in *g(23,4)*, and we categorize her as an equilibrium second-mover with *IPOR* = 1 (if 18 was the earliest equilibrium position available to this player). In this instance two possible things may have happened. Either a second-mover had realized the equilibrium play all along but simply did not get an opportunity to secure any equilibrium position other than the last two winning positions, or, she simply had not identified the equilibrium play path and just stumbled onto those two positions without realizing that those are the equilibrium ones. In this case we will overestimate our measure of the equilibrium second-mover by including her as an equilibrium player. A little reflection however makes it clear that it may be quite impossible to come up with a measure that does not suffer from some level of imperfection while attempting to classify a second-mover as an equilibrium player. In case of first-movers however there remains no such ambiguity since the chance is extremely low that a first-mover would simply stumble onto all the equilibrium positions. In order to remedy this limitation, we will focus on the aggregate data while statistically testing the extent of second-movers' equilibrium play across treatments. We hope that by doing so we can level off some of the noise due to imperfection.

We now turn our attention to how a change in the l of a Game of Position (and holding the n constant) affects the frequency of the equilibrium play by the second-movers. The evidence is in Table 4. The second row of the table reports, for each treatment, how many 'first-ever' opportunities second-movers had obtained to potentially occupy all the remaining equilibrium positions in a game and out of that how many times they actually occupied all those positions from that point beyond [18]. Following this row from left to right one can deduce that such opportunities increased in sheer magnitude (from 80 to 142) as playing the dominant-strategy for the first-movers presumably became increasingly (cognitively) difficult.

Did altering the l and keeping the n fixed influence second-movers' equilibrium play? The answer is yes according to the second row of table 4. The aggregate data (row two) indicate that the second-movers were able to compute at least one step of reasoning and win a game in 42/80 (53%) instances in *g(15,4)*, whereas in *g(23,4)* the percentage drops to 42 (62/148), indicating a reduction (11%) in the equilibrium second-movers. The reduction (16%) is more pronounced between the treatments *g(15,3)* and *g(23,3)*. A statistical analysis of the aggregate data confirms that increasing the l (and keeping the n constant) have adversely affected the performance of the equilibrium second-movers in both cases[19].

However the conclusion drawn in the preceding paragraph definitely suffers from some imprecision due to the chance factor. In the above aggregate analysis we have added up all types of equilibrium second-movers (*i.e.*, $1 \leq IPOR \leq m$) in a treatment to arrive at that conclusion. But a second-mover with $1 \leq IPOR < m$ in a treatment may actually have played so by mere chance and for no clever reason whatsoever. In that case, our conclusion about the impact of increased l on the proportion of the equilibrium

[18] To explain how we obtain these numbers, let us consider an example. In *g(15,4)*, say, a first-mover lands on position 4 (thus not playing the dominant-strategy), then the second-mover can play the dominant-strategy from that point onwards by computing two *SOR* and win the game. In our analysis this is counted as one opportunity for a second-mover to play the dominant-strategy. We do not consider this game again to detect a lower *SOR* opportunity for the same second-mover.

[19] The difference in the proportion of second-movers computing at least one step of reasoning in *g(15,4)* and the proportion of second-movers computing at least one step of reasoning in *g(23,4)* is significant (Z statistic = 1.45, $p = 0.07$). The difference in the proportion of second-movers computing at least one step of reasoning in *g(15,3)* and the proportion of second-movers computing at least one step of reasoning in *g(23,3)* is highly significant (Z statistic = 2.32, $p = 0.01$).

second-movers is somewhat misleading. In order to allay this concern we conducted an additional analysis. We consider the proportion of those second-movers in each treatment who, just like an equilibrium first-mover in a game, played the dominant-strategy by computing all the *SOR* in a game (*i.e.*, *IPOR* = *m* for these equilibrium second-movers). This way of analysis avoids the above-mentioned ambiguity and helps us answer the question in the most possible clear-cut manner given our data.

In *g(15,4)* when *IPOR* = *m* = 2, 26 of 64 (41%) games correspond to the dominant-strategy play by the second-movers whereas in *g(23,4)* when *IPOR* = *m* = 4, 4 of 14 (29%) games correspond to the dominant-strategy play by the second-movers. However between the treatments *g(15,3)* and *g(23,3)*, the occurrence of the dominant-strategy play by the second-movers drops dramatically from 50 percent to 24 percent[20]. *This gives our third main result that an increase in the l of a Game of Position, while holding the n fixed, leads to a statistically significant reduction in the proportion of the second-movers computing at least one step of reasoning (from the very end of the game tree) when we consider the aggregate data. The same increase in the l leads to a statistically significant reduction in the incidence of the dominant-strategy play by the second-movers for the treatment pair g(15,3) and g(23,3) when we consider the IPOR = m data, but not so for the treatment pair g(15,4) and g(23,4).*

Did altering the *n* and keeping the *l* fixed influence second-movers' equilibrium play? When we look at the second row of table 4, *i.e.*, considering the aggregate play data, we find that the proportion of the equilibrium second-movers, computing at least

[20] *IPOR = m data*: The difference in the proportion of second-movers playing the dominant-strategy in *g(15,4)* and the proportion of second-movers playing the dominant-strategy in *g(23,4)* is not significant (Z statistic = 0.53, $p = 0.3$). The difference in the proportion of second-movers playing the dominant-strategy in *g(15,3)* and the proportion of second-movers playing the dominant-strategy in *g(23,3)* is highly significant (Z statistic = 2.26, $p = 0.02$).

one step of reasoning, does not change between *g(15,4)* and *g(15,3)*. The proportion decreases from 42 percent in *g(23,4)* to 37 percent in *g(23,3)*. When we consider the proportion of those second-movers with *IPOR* = *m*, we find that the proportion of this type of equilibrium second-movers increases from 41 percent in *g(15,4)* to 50 percent in *g(15,3)*. The proportion decreases from 29 percent in *g(23,4)* to 24 percent in *g(23,3)* [21]. *This gives our fourth main result that a decrease in the n of a Game of Position, while holding the l fixed, does not lead to any statistically significant difference in the incidence of the equilibrium play by the second-movers for any treatment pair when we consider either the IPOR = m data or the aggregate data.*

To sum up, the above analyses uncover that the dominant-strategy play is negatively correlated with the *l* of a Game of Position (holding the *n* fixed); however the effect of decreasing the *n* (holding the *l* fixed) of a Game of Position on the optimal play is somewhat unambiguous. A decrease in *n* lowers the frequency of the dominant-strategy play between the shorter planning horizon games, but leads to a rise in the dominant-strategy play between the longer planning horizon games. These results seem to suggest that in general limiting cognitive constraint may help achieve higher proportion of the dominant-strategy play.

4.2 Learning

The previous analyses of the aggregate data seem to suggest that subjects in our experiment might have learnt to play the dominant-strategy as play progressed. In order to focus on how learning about the dominant-strategy play occurs in each treatment, we concentrate on the time series of the aggregate and individual play data in this subsection.

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[21] Again we conducted the above two types of statistical tests, one using the aggregate data and the other using the data with *IPOR* = *m* for the second-movers. None of the tests came out to be significant at usual levels of significance.

We start this analysis with the evolution of the dominant-strategy play. Figure 2 presents the time series of the proportion of the equilibrium first-movers in each treatment. A casual inspection reveals that overall incidence of the dominant-strategy play is higher in treatments with fewer *SOR*. In the very first period, when subjects did not have any prior experience, 25 percent first-movers played according to the dominant-strategy in *g(15,4)*, while corresponding figures are 13, 0, and 0 for *g(15,3)*, *g(23,4)*, and *g(23,3)*, respectively. With experience, the proportion of the equilibrium first-movers increased in all the treatments, however at different rates. In the final period, the proportion of the equilibrium first-movers is 0.88 in *g(15,4)* and *g(23,3)*, and the corresponding figures are 1 and 0.5 in *g(15,3)* and *g(23,4)* respectively. As expected, the winners in later periods were primarily determined by the assignment of the first-mover role in all the treatments except in *g(23,4)*. While the figure provides enough evidence that first-movers in a given treatment played the dominant-strategy with greater frequency with the repetition of the stage game, it does not offer any information as to what proportion of subjects in a given treatment actually learnt the dominant-strategy play. We take up this issue next.

The observed difference in the incidence of the equilibrium first-movers in later periods in *g(23,4)* as compared to the other three treatments may arise due to two reasons. First, a larger proportion of the subject population in *g(15,4)*, *g(15,3)*, and *g(23,3)* may have learnt to play the dominant-strategy as compared to *g(23,4)*. Second, even if the proportion of the subject population who had learnt to play the dominant-strategy remains the same in each treatment, the subjects who had learnt to play the dominant-strategy may have happened to get less opportunities to play as first-movers in *g(23,4)* than in *g(15,4)*, *g(15,3)*, and *g(23,3)*. The following analysis sheds light on this issue.

Figure 3 presents the frequency distribution of players as the first-mover in all four treatments. Note that a player can get zero to 15 opportunities to play the role of a first-mover in a treatment as each session had 15 periods and the computer assigned the role of a first-mover in a random manner. The empirical distribution as represented in figure 3 closely follows a theoretical Binomial distribution with a mean equal to 7.5 and a variance equal to 3.75. The realized mean of first-mover opportunities is exactly equal to 7.5 in each treatment and the highest frequency in each treatment concentrates in the interval of 5-9[22]. Therefore the distribution of the role-assignment did not differ markedly across the treatments. In other words, the number of times a subject was chosen a first-mover did not differ much on average across treatments. Hence we conclude that it cannot be the case that on average the subjects who had learnt to play the dominant-strategy may have happened to get considerably fewer opportunities to play as first-movers in one treatment as compared to another. Next we investigate whether the proportion of the subjects who learnt to play the dominant-strategy differs considerably across treatments, which may explain the observed difference in the incidence of the equilibrium first-movers in later periods between *g(23,4)* relative to the other three treatments.

Figure 4 presents, for each treatment, the cumulative distribution of players across time who had played according to the dominant-strategy prediction (both mover-types

[22] We also conducted the same analysis for the last eight periods in which the theoretical Binomial distribution has a mean equal to 4 and a variance equal to 2. The mean first-mover opportunity is exactly equal to 4 in each treatment and the highest frequency in each treatment concentrates on the interval of 3-5 in this case.

with *IPOR* = *m*) at least once. The difference between the proportions in period (*t+1*) and *t* in a given treatment thus reflects the addition of players to the pool of existing players who have already played the optimal strategy at least once.

In the first period when no element of experience exists, the dominant-strategy play is the highest in *g(15,4)* and the lowest in *g(23,3)*. By the final period, the proportion of players who had played according to the dominant-strategy prediction at least once is the highest in *g(15,3)*, followed by *g(15,4)*, *g(23,3)*, and *g(23,4)*. In the last period, this proportion is 0.56 in *g(23,4)* whereas the numbers are considerably higher in the other three treatments: 1, 0.94, and 0.81 for *g(15,3)*, *g(15,4)*, and *g(23,3)*, respectively. These numbers therefore shed some light on why *g(23,4)* may have emerged *ex post* the most difficult game in our experiment. The cumulative distributions corresponding to the treatments with smaller *l* lie strictly above that of the treatments with larger *l*. The data therefore suggest that greater cognitive burden imposed by relatively longer planning horizons may have reduced the proportion of such players. Furthermore, the cumulative distributions corresponding to the treatments with smaller *n* lie strictly above that of the treatments with larger *n* (controlling for *l*) in later periods. More importantly, this difference is strikingly higher for the pair *g(23,3)* and *g(23,4)* than for the pair *g(15,3)* and *g(15,4)* in last few periods. This extends some support to our earlier conjecture that reducing the *n* (holding *l* fixed) may have eased a subject's cognitive burden more in longer decision-tree games.

While the previous analyses are suggestive of the possibility that subjects gradually learnt the dominant-strategy play in each treatment and this learning differed across treatments, they however fall short of identifying the actual learning patterns that have emerged in each treatment. In order to gain further insights into whether and how players actually learnt about the dominant-strategy play in each treatment, we conducted two types of analyses.

First, we consider the time series of the distribution of *IPOR* in each treatment. Figure 5-8 represent the time series for $g(15,4)$, $g(15,3)$, $g(23,4)$, and $g(23,3)$, respectively[23]. This will inform us whether at an aggregate level subjects had learnt to compute higher number of *SOR* as a treatment advanced. That is, on average whether subjects learnt to compute the very first step of reasoning (from the very end) at the beginning, then the first two *SOR*, and so on, and finally the entire chain of the reasoning process in a given treatment. If this were the case, then we would expect the occurrences of the higher-valued *IPOR* to increase over time in a given treatment.

In the first period of each treatment with the exception of $g(15,4)$, the proportion of $IPOR = m$ is strictly lower and the proportions of lower-valued indices are considerably higher, indicating that initially a greater fraction of subjects in a given treatment may have found it cognitively difficult to carry out all the *SOR*. In contrast, in $g(15,4)$ the shares of $IPOR = m$ and $0 \leq IPOR < m$ are exactly equal in the first period. This indicates that subjects in this treatment have found it the easiest to figure out the optimal play relative to other treatments. The incidence of play governed by $IPOR = m$ increased steadily in $g(15,4)$, $g(15,3)$, and $g(23,3)$ in later periods, while this trend is not so sharp in $g(23,4)$ in which even in the last period only 50 percent of all games correspond to a play governed by $IPOR = m$. The consistent increase in the incidence of

[23] Note that this analysis pools the data for both types of movers.

$IPOR = m$ with progress of play in all the treatments provides evidence that on an aggregate basis subjects gradually succeeded in figuring out the dominant-strategy play in each treatment as opposed to just occupying the equilibrium positions at random. Furthermore, this success rate differs considerably between *g(23,4)* and the other three treatments. We also observe that the occurrences of the non-equilibrium play ($IPOR < m$) exhibit a faster trend of decay in treatments with $l = 15$ than in treatments with $l = 23$. This decline is the fastest in *g(15,4)* and the slowest in *g(23,4)*. This accords well with out previous finding that the fractions of subjects who played the dominant-strategy play at least once are smaller in games with $l = 23$ than in games with $l = 15$. Overall, these findings indicate that initially many subjects were unable to detect the full chain of reasoning required for the dominant-strategy play and instead could compute only last few *SOR* that are needed to win a game. Alternatively, they may have been entirely clueless about the dominant-strategy play and plausibly adopted a forward-looking approach.

While the evolution of the *IPOR* in each treatment provides clear indication of the learning of the dominant-strategy play at an aggregate level (*i.e.*, on average subjects simply did not stumble onto the equilibrium positions), it is still unclear from these analyses whether an individual subject really learnt the dominant-strategy play in each treatment. One way of detecting individual learning of the optimal play is to check if a subject who had played the dominant-strategy in period t ever faltered to play optimally in subsequent periods. If a subject simply stumbled on to the equilibrium positions and had no clue about the dominant-strategy play, then a subject playing the equilibrium strategy in t may very well fail to play the equilibrium strategy in period $(t + j)$, if she is faced with an opportunity to play the dominant-strategy in period ($t + j$). But if a subject occupied all the equilibrium positions in period t and also understood the optimal strategy, then she would never miss a future opportunity to play according to the optimal strategy regardless of whether all the *SOR* or a few of them remain to be computed.

Figure 9 presents the distribution of the individual errors as a proportion of actual opportunities where an error is defined as: after a subject i attained her first $IPOR = m$, that subject i either missed an opportunity to play the dominant-strategy in the role of a first-mover in any subsequent period or missed an opportunity of 'equilibrium play' as a second-mover in any subsequent period[24]. For each subject i, we express the total number of such errors as a proportion of the total number of such opportunities. The figure displays the distribution of such proportions for all subjects in a given treatment. In this analysis, we do not include a player who has already attained her first $IPOR = m$, but did not get any opportunity of equilibrium play in further periods because she was always chosen as the second-mover in these later periods and her opponents had always played the dominant-strategy on those occasions. We also do not include the second-movers with $IPOR = 0$ in this analysis.

The figure reveals two important behavioral regularities about individual learning of the dominant-strategy play. First, in all treatments except *g(15,3)* the error value zero achieves the highest frequency. 88 percent of subjects in *g(23,4)* who ever played the dominant-strategy in a previous period did not falter to play the equilibrium strategy in subsequent periods whenever such opportunities came on her way. The same percentages are 67, 67, and 13 in *g(23,3)*, *g(15,4)*, and *g(15,3)*, respectively. Therefore the first

[24] Recall that equilibrium play for a second-mover implies that the player did not make any mistake in occupying the equilibrium positions from the earliest position available to her in a game.

occurrence of *IPOR* = *m* provides a good indicator of individual learning of the dominant-strategy play in all three treatments except in *g(15,3)*[25]. Second, even though relatively smaller fractions of subjects played the dominant-strategy at least once in games with $l = 23$ than that of in games with $l = 15$ (as indicated by the final period data of Figure 4), it appears that the first occurrence of *IPOR* = *m* is a better indicator of individual learning of the dominant-strategy play in games with $l = 23$ than in games with $l = 15$ as the proportion of players with zero subsequent errors in equilibrium plays is much higher in games with larger l.

To sum up, the observed learning trends indicate that experience helped in boosting the equilibrium play in all the treatments. The aggregate data from all four treatments indicate that subjects were able to compute few *SOR* in the initial periods, but gradually learnt to carry out higher *SOR*. The data on individual subjects provide substantial evidence that once subjects play in accordance with the optimal strategy; they mostly adhere to that play from that point onwards.

5. Conclusions

Most optimal economic decisions presume substantial amount of rationality, which in turn calls for sufficient cognitive power. However human cognition is not limitless. A voluminous literature hitherto has demonstrated that beyond a certain level of complication, humans' logical machinery ceases to function – a sign of bounded rationality. How this limited cognitive capacity, represented by finite depth of reasoning, interacts with economic decisions and influences quality of economic outcomes is a question of immense importance to economists.

[25] We do not have any well grounded explanation for this observed random pattern of individual play in *g(15,3)*.

This paper experimentally studies a two-player two-outcome strictly competitive game with perfect information to explore the above issue. We call this the Game of Position that is characterized by a first-mover advantage and a dominant-strategy solution. The equilibrium strategy involves a multi-stage iterative reasoning procedure and is independent of players' beliefs about each other. Hence the structure of the game is especially favorable for a pure and in-depth examination of reasoning capacity of individual players. Alongside this, the game captures essential aspects of many important economic decisions like financial planning for retirement, working towards a degree, working for a targeted weight reduction etc. that also require multi-stage decisions consisting of a series of interdependent stages leading towards a final outcome. Hence the game offers an ideal environment in which we can test the obvious connection between human reasoning-skill and quality of economic outcome.

The Game of Position is defined by two parameters, the length (l) of the decision tree and the maximum steps (n). These parameters jointly determine how many steps of reasoning a player must compute to play in accordance with the dominant-strategy. We generate four Games of Position by altering l and n, and thereby we vary the cognitive demands on players in a systematic manner, and assess its impact on the dominant-strategy play. Conditional on our maintained assumption that figuring out increasing number of steps of reasoning commands higher cognitive effort by players, the laboratory scrutiny of each game therefore enables us to directly observe whether the incidence of the optimal play declines as reasoning process becomes increasingly complicated across the games.

The main result is that the frequency of the dominant-strategy play sharply increases as we limit the cognitive demand on players. The data indicate that the incidence of the equilibrium play is inversely related to the l (holding n constant), but the effect of varying the n (holding l constant) on the dominant-strategy play is not unambiguous. The analysis of the equilibrium play data thus implies that an optimal multi-stage planning process can be severely impeded by the intricacy of the task-related reasoning procedure. Our analysis also sheds light on subjects' learning pattern of the equilibrium play. The results show that initially many subjects won a game by computing only the last few steps of the sequential reasoning process, but after gaining enough experience majority of them learn to decipher additional steps of reasoning and thus play the dominant-strategy. We conjecture that this pattern of play may arise due to subjects' inability to carry out all the steps of reasoning except the last few steps in initial stages of the play, or may be due to subjects' initial tendency to play according to an ad-hoc forward-looking approach. Thus learning the optimal strategy by gradual experimentation is another way to think of our results.

How optimal decision in a game is closely intertwined with players' cognitive capacity players is an important issue for economists. Laboratory techniques may be immensely helpful in revealing processes by which players make certain payoff-maximizing decisions in games. This is an important line of research because economic outcome is mostly governed by players' behavior and this behavior in turn is controlled by human cognition. Yet, only recently have economists begun to take interest in this obvious connection between human mind and economic decisions. Our research may be thought of as a small step towards understanding this relationship.

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Figures

Figure 1. The equilibrium play in the Game of Position *g(23,3)*

Position	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
Decision			**X**				**X**				**X**				**X**				**X**				**X**

Note: **X** *denotes an equilibrium position for a first-mover*.

Figure 2. Time series of proportion of the equilibrium first-movers by treatment

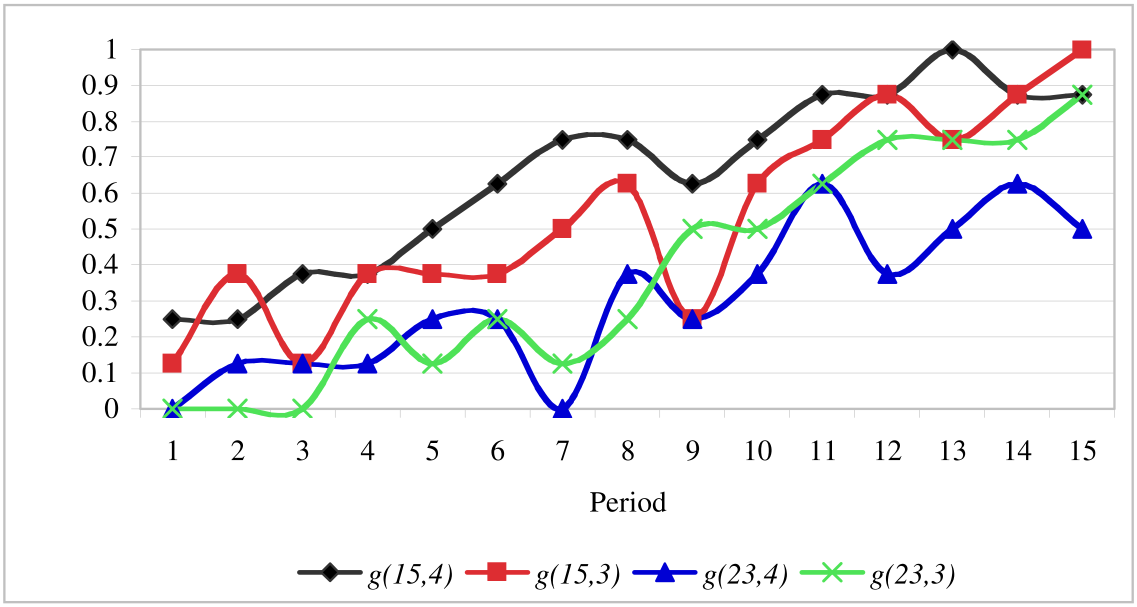


Figure 3. Frequency distribution of players in the role of first-mover by treatment

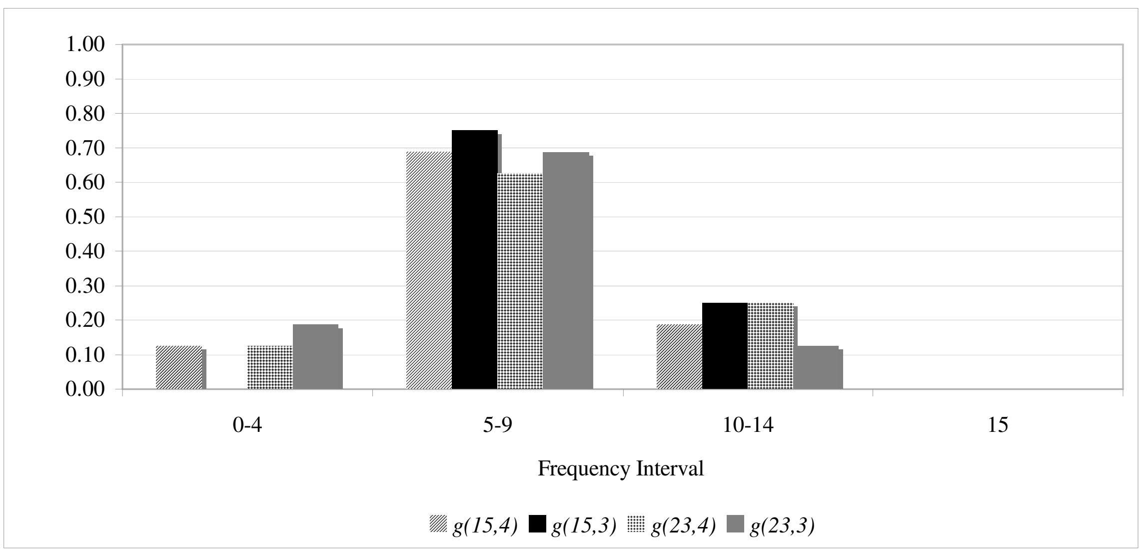


Figure 4. Cumulative distribution of players characterized by *IPOR* = *m* at least once by treatment

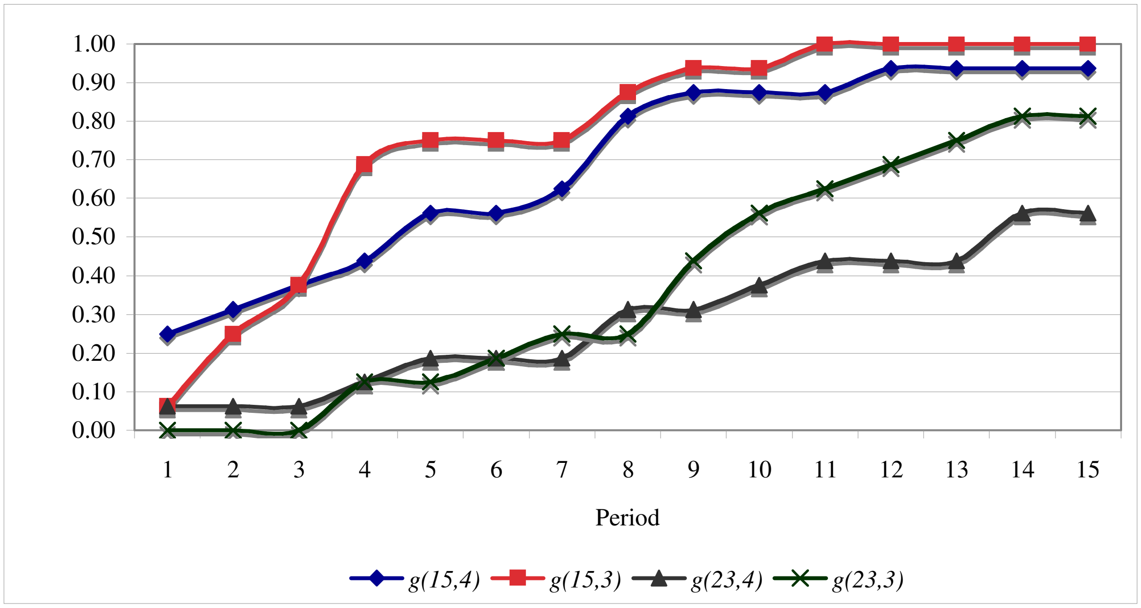


Figure 5. Distribution of *IPOR* in *g(15,4)*

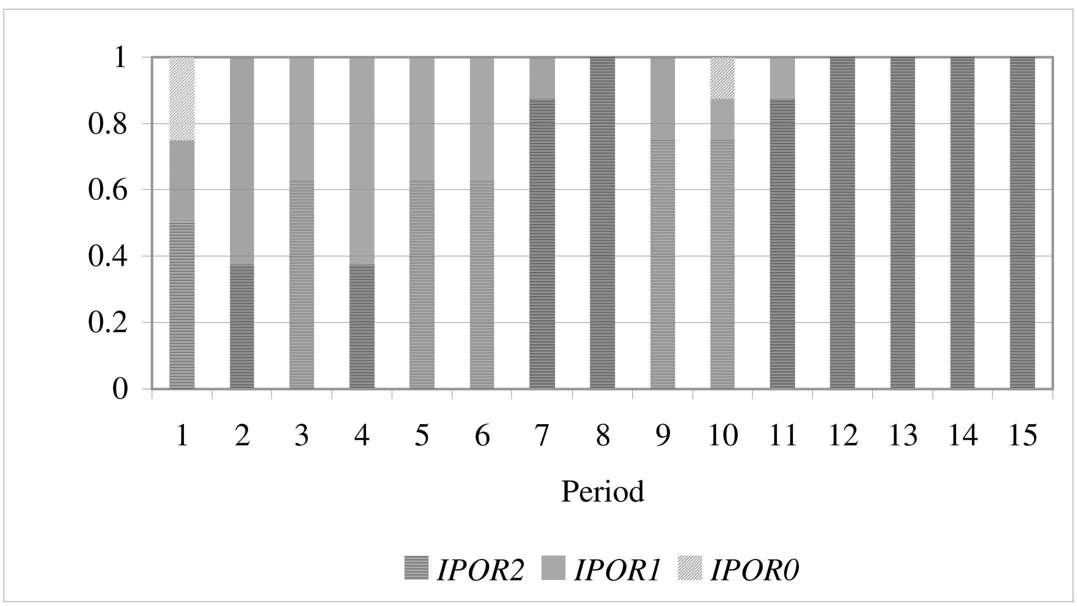


Figure 6. Distribution of *IPOR* in *g(15,3)*

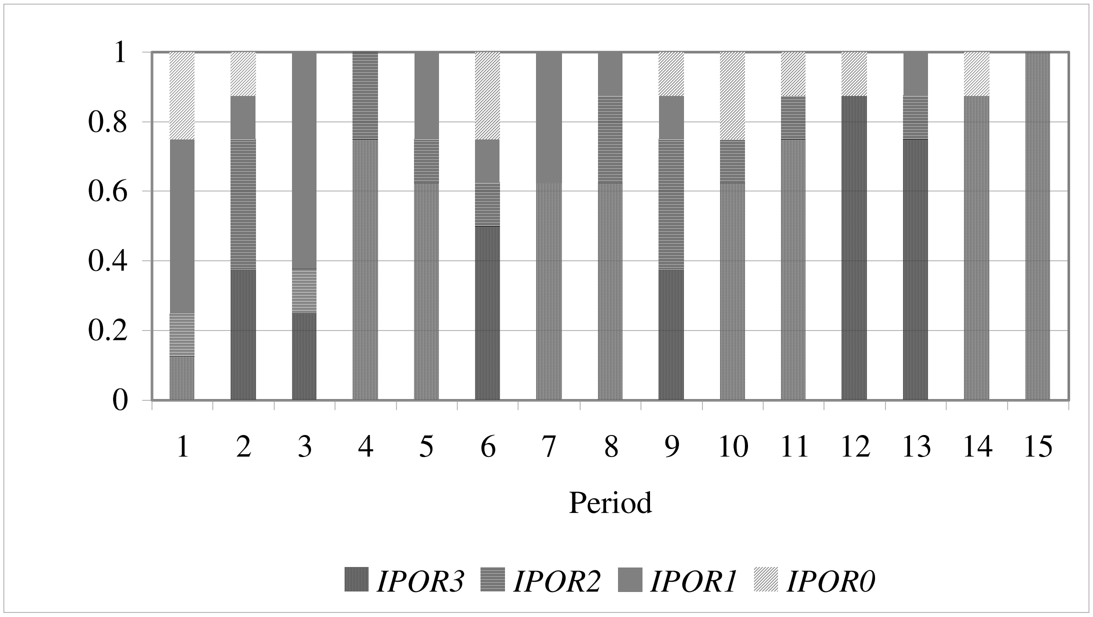


Figure 7. Distribution of *IPOR* in *g(23,4)*

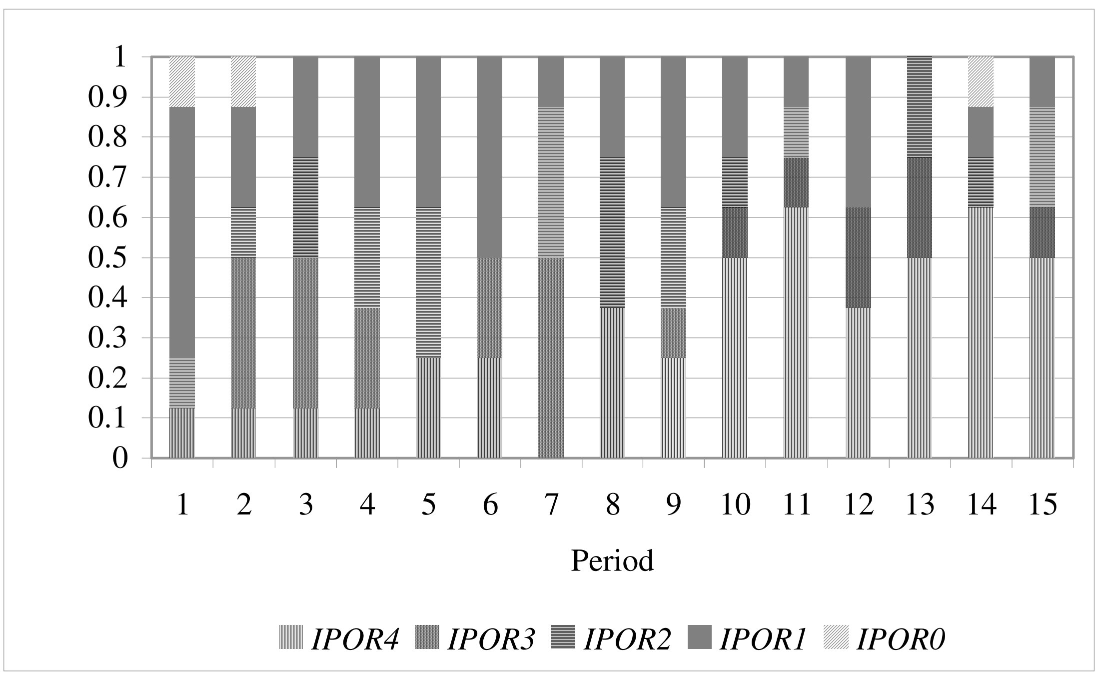


Figure 8. Distribution of *IPOR*s in *g(23,3)*

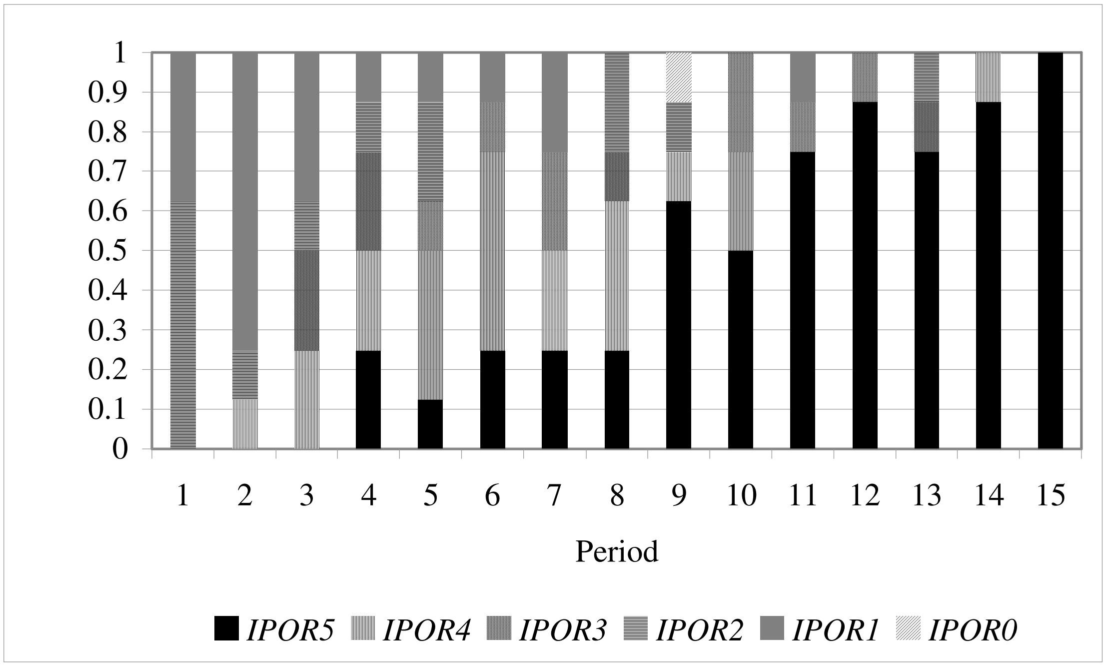
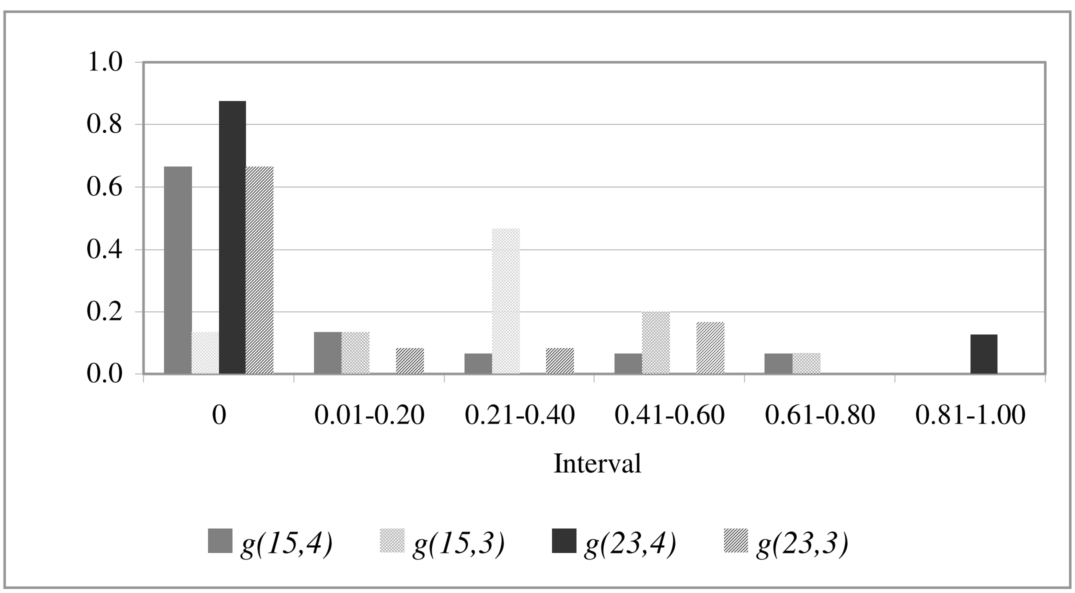


Figure 9. Distribution of errors as proportion of opportunities after the occurrence of first *IPOR =m*



Tables

Table 1. Number of equilibrium positions (number of *SOR*) by game-type

Maximum Steps (*n*)		***4***	***3***
Decision Tree Length (*l*)	***15***	3 (*2*)	4 (*3*)
	23	5 (*4*)	6 (*5*)

Note: Figures in parentheses denote the number of SOR associated with the dominant-strategy

Table 2. Distribution of games won by each mover-type

Treatment ➔ Mover-type 🡇	***g*(*15,4*)**	***g*(*15,3*)**	***g*(*23,4*)**	***g*(*23,3*)**
First-Mover	178 *(0.74)*	150 *(0.63)*	148 *(0.62)*	148 *(0.62)*
Second-Mover	62 *(0.26)*	90 *(0.37)*	92 *(0.38)*	92 *(0.38)*
Total Games	240	240	240	240

Note: Figures in parentheses denote proportion of the total games

Table 3. Dominant-strategy play by the first-movers

Treatment ➔	***g*(*15,4*)**	***g*(*15,3*)**	***g*(*23,4*)**	***g*(*23,3*)**
The Dominant-Strategy Games	156 (*0.65*)	128 (*0.53*)	72 (*0.3*)	92 (*0.38*)
Total Games	240	240	240	240

Note: Figures in parentheses denote proportion of the total games

Table 4. Distribution of equilibrium second-mover *IPOR*s by treatment

Treatment ➔	***g*(*15,4*)**	***g*(*15,3*)**	***g*(*23,4*)**	***g*(*23,3*)**
All *IPOR*s	42/80 (*0.53*)	52/98 (*0.53*)	62/148 (*0.42*)	52/142 (*0.37*)
***IPOR* = 1**	16/16 (*1.00*)	10/22 (*0.45*)	4/8 (*0.5*)	2/2 (*1.00*)
***IPOR* = 2**	26/64 (*0.41*)	24/40 (*0.60*)	16/40 (*0.4*)	4/4 (*1.00*)
***IPOR* = 3**	-	18/36 (*0.50*)	38/86 (*0.44*)	8/32 (*0.25*)
***IPOR* = 4**	-	-	4/14 (*0.29*)	26/54 (*0.48*)
***IPOR* = 5**	-	-	-	12/50 (*0.24*)

Note: Figures in parentheses denote proportion of all the feasible opportunities received by second-movers for that IPOR type