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Perfect Price Discrimination is not So Perfect

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Abstract

The foundation of the accepted theory on two-part tariffs is the partial equilibrium analysis first developed by Oi (1971). He argues that the profit maximum obtains from a lump-sum payment (equal to the consumer surplus) plus a unit price (equal to marginal cost), and that the resulting allocation is Pareto efficient because it is identical to perfect competition (except for lump-sum transfers to the monopoly). He shows that this outcome is identical to first-degree price discrimination. This analysis is widely included in undergraduate and graduate level textbooks, and is often cited as a basis for the public regulation of utilities. A few general equilibrium papers also validate Oi's partial equilibrium conclusion. By contrast, we present a general equilibrium counterexample that shows that this conventional conclusion cannot be generally correct.

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1. Introduction

Conventional wisdom holds that perfect price discrimination does not represent a loss of economic efficiency. By redistributing income in favor of the monopolist it does pose a social equity problem, but this issue is thought to be separable from efficiency. Thus, economic theory encourages policymakers to combine, as independent agendas, social safety net policies and price discrimination in industries with natural market power. This paper questions the generality of this conventional wisdom.

Our objection is that the standard partial equilibrium argument for price discrimination cannot be generalized, except under special conditions. For example, price discrimination is efficient when production costs must be zero, as with the Cournot natural spring. The zero cost case is inconsistent with the usual partial equilibrium story.

We review the literature on first-degree price discrimination and the two-part tariff in the next section. The following sections develop a series of examples. We begin with a one-agent economy, two-goods, and quasilinear preferences. We show that, contrary to conventional wisdom, the general equilibrium for this case results in a different allocation in the competitive and price discrimination regimes; the former is efficient, while the latter is not. We explore other examples, demonstrating that price discrimination is also inefficient with decreasing returns to scale (Section 4). Section 5 explores the consequences of generalizing our quasilinear counterexample to multi-agent economies. We find that as long as all agents own equal shares of the monopoly the one-agent conclusions are unchanged. However, when ownership is highly unequal, counterintuitive corner solutions can occur that are Pareto inferior to regimes with greater equality.

2. Literature review

The academic literature on two-part tariffs and first-degree price discrimination[1] and the most widely used microeconomics textbooks[2] all argue that a monopoly that practices first-degree price discrimination is efficient. The literature is summarized in Table 1.

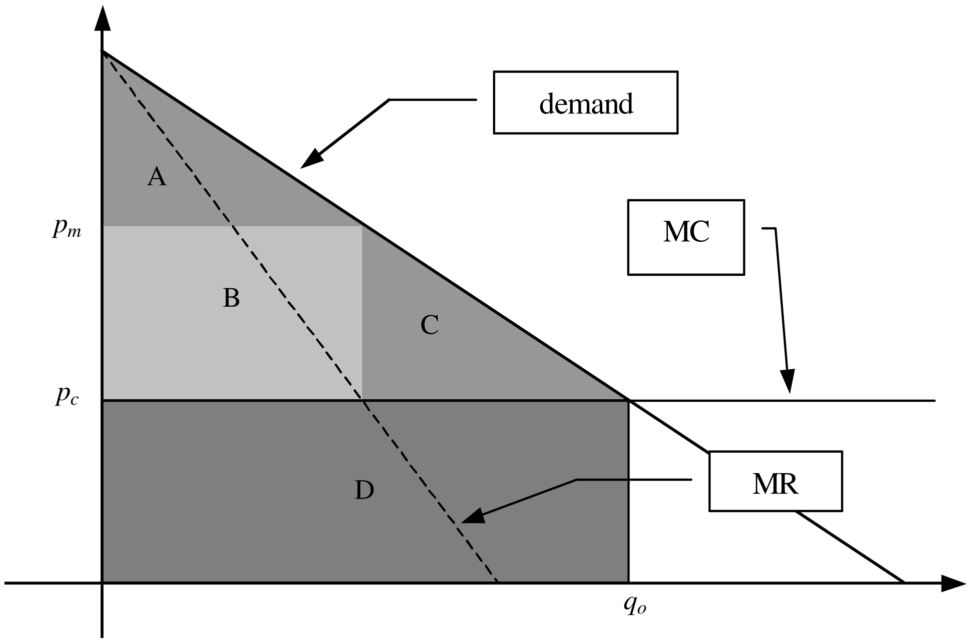
[1] See Oi (1971), Braeutigam (1989), Phlips (1983), Norman (1999), and Varian (1989).
[2] See Pindyck & Rubinfeld (1998), Varian (1992) and Nicholson (1998).

Table 1. Literature Summary

Author(s)	Model	Equilibrium at $p=MC$	Pareto efficient
Braeutigam (1989)	partial	yes	implicit
Braverman, Guasch and Salop (1983)	partial	price is above MC due to the presence of defective units	implicit
Brown and Heal (1980)	general	contingent on whether fixed parts based on individual preferences and associate with purchase of particular goods	sometimes
Feldstein (1972)	partial	contingent on variations in income distribution and demand elasticity	sometimes
Kolay and Shaffer (2003)	partial	sometimes, depending on whether the consumer is high-demand or low-demand for a two-part tariff	implicit
Leland and Meyer (1976)	partial	sometimes	sometimes
Littlechild (1975)	partial	yes	implicit
Naughton (1989)	partial	no, due to differences in preference rates and consumption levels	implicit
Ng and Weisser (1974)	general	yes, except under extreme demand conditions	sometimes
Nicholson (1998)	partial	yes	implicit
Norman (1999)	partial	yes, given observable consumer preferences	yes
Oi (1971)	partial	yes	yes
Phlips (1983)	partial	yes	yes
Pindyck and Rubinfeld (1998)	partial	yes	implicit
Schmalensee (1918)	partial	contingent on deviations of marginal demand from average demand	implicit
Shaffer (1986)	partial	yes	yes
Shaffer (1992)	partial	yes, unless there is uncertainty or interdependent demand	sometimes
Tirole (1988)	partial	yes	yes
Varian (1989)	partial	yes	yes
Varian (1992)	partial	yes	yes
Willig (1978)	partial	yes; if not, then nonlinear outlay schedules Pareto dominate	yes

Typically this conclusion is drawn from the partial equilibrium argument illustrated in Figure 1. For the case of the price discriminating monopolist, the profit-maximizing strategy is a set of individualized take-it-or-leave-it offers to consumers that add up to the sum of areas A through D in exchange for a quantity q_o. While for the monopolist who can impose a two-part tariff offers to sell her output at p_c after first charging entry fees that add up areas A+B+C. The conventional welfare implication derives from the observation that the quantity supplied in both cases is identical to that of perfect competition. In either case the resulting profit is A+B+C, which exceeds the smaller profit B of the monopoly price p_m. Phlips (1983) writes, "if a firm has the possibility of discriminating, perfect discrimination is the best type of discrimination to use."

Figure 1. Monopoly pricing options in partial equilibrium



Most of the literature regarding two-part tariffs stems from Oi's (1971) partial equilibrium analysis. His analysis is essentially that of Figure 1. He favors the two-part tariff because price equals marginal cost, and because the outcome is efficient. About half of Table 1 are simple restatements of Oi.[3] Others extend the partial equilibrium analysis in one way or another.[4]

Willig (1978) extends Oi's analysis showing that whenever price exceeds marginal cost, both consumers and firms prefer some nonlinear outlay schedule (such as a two-part tariff) with price equal to marginal cost. Littlechild (1973) extends marginal-cost pricing to the case of consumption externalities. Naughton (1989) extends the analysis by specifying the preferences for the governmental regulator. He investigates the possibility that regulator's objective function may reveal different weights among producers, consumers, or different income groups. A unique paper by Braverman, Guasch and Salop (1983) argues that two-part pricing can be inefficient when some of the monopoly-supplied goods are defective.

It would be prudent to confirm Oi's fundamental policy conclusion with a general equilibrium analysis, but only two papers listed in Table 1, Brown and Heal (1980) and Ng and Weisser (1974), do so. Brown and Heal argue that marginal-cost pricing is efficient based on a sophisticated topological proof from an earlier unpublished paper. However, the later paper does not acknowledge that the former proof is limited to consumers with linearly homogeneous preferences. Since the demand curve invoked in Figure 1 can only be derived from non-homogeneous preferences, the relevance of their result is questionable.

3. A Counterexample

A two-good economy with a single agent

Partial equilibrium analysis ignores any consequences of changes in income or price in other markets. Most important are any effects in the markets for the factors used to produce the monopolist's

[3] These include Nicholson (1998), Norman (1999), Phlips (1983), Pindyck and Rubinfeld (1998), Shaffer (1986), Varian (1989), and Varian (1992).

[4] These include Braeutigam (1989), Braverman Guasch and Salop (1983), Feldstein (1972), Kolay and Shaffer (2003), Leland and Meyer (1976), Littlechild (1975), Naughton (1989), Schmalensee (1981), Shaffer (1992), and Willig (1978).

output. We illustrate this point with an example in which marginal-cost pricing is efficient, but where a monopolist with the power to price discriminate will chose a pricing policy which is inefficient.

Consider an economy with only one agent, and two goods, x_1 and x_2. The agent has an endowment ω of the first good. Her non-homogeneous utility takes the quasilinear form $U(x_1, x_2) = x_1 + u(x_2)$, where $u' > 0$ and $u'' < 0$.

y's denote the same two goods on the supply side. The first good is sold as a productive input $y_1 = x_1 - \omega$, which is always negative. y_2 is produced from y_1 according to production function $y_2 = f(y_1)$, where f is twice differentiable and $f' < 0$. $f'' \leq 0$; it may be zero (as in the constant returns to scale examples discussed below), or negative for decreasing returns. Supply behavior may not be well-behaved in cases of increasing returns. It is also assumed that f is invertible for negative values of y_1. The agent owns the firm.

First consider a competitive market where the agent is a price taker in both consumption and production decisions.[5] Taking the price of the first good as the numeraire (p_1=1), the firm's profit is $\Pi = p_2 y_2 + y_1$ (remember that y_1 is negative). Substituting the inverse of the production relation, recognizing that $y_2 = x_2$ in equilibrium, and maximizing gives the supply curve as $p_2 = -\frac{df^{-1}}{dx_2}$.[6] This economy's transformation function can be written as $0 = \omega - x_1 + f^{-1}(x_2)$.

Substituting the budget constraint into utility, gives

$$u(x_1, x_2) = (m - p_2 x_2) + u(x_2),$$

where m is her income, her endowment plus any profits earned by the firm. Thus, the demand curve for the second good is $\frac{du}{dx_2} = p_2$. This is a well-behaved demand curve and usually a competitive equilibrium exists where $\frac{du}{dx_2} = -\frac{df^{-1}}{dx_2}$, as long as $\frac{d^2u}{dx_2^2} < \frac{d^2 f^{-1}}{dx_2^2}$. This is Pareto efficient by the First Theorem of Welfare Economics.

With 1st-degree price discrimination each consumer is offered a take-it-or-leave-it deal: (r, x_1, x_2) or $(0, \omega, 0)$. The largest payment r is given by

$$x_1 + u(x_2) - r = \omega + u(0)$$

Varian (1992) also uses this approach, arriving at the conventional efficiency result. But he writes the payment as $U(0, x_2) - r_v = U(\omega, 0)$. This definition ignores the fact that x_2 must be produced by from x_1. In defense of Varian's formulation it might be argued that the firm's offer is (r, x_2), ignoring input requirements, and that the price-taking consumer is unaware of the firm's production function. To do so is myopic however; the myopic consumer would discover later that x_1 is not the same under both options. One case in which these two formulations are identical occurs when production is costless, as in Cournot-spring examples.

Adopting the general equilibrium approach with a take-it-or-leave-it offer of (r, x_1, x_2), the monopolist's profit is

$$\Pi = r + y_1 = x_1 + u(x_2) - \omega - u(0) + y_1.$$

Substituting for the payment and the transformation function gives

$$\Pi = u(x_2) - u(0) + 2f^{-1}(x_2).$$

The first order condition is $\frac{du}{dx_2} = -2\frac{df^{-1}}{dx_2}$. The price discrimination equilibrium is inefficient since it differs from the competitive one. Thus, discrimination produces too little x_2.

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[5] The price-taking assumption appears to be implausible or the one-agent economy. We defend it by a large-economy generalization in Section 5 where there are many agents. If one agent is the sole owner of the monopoly, she would easily be able to ignore her personal consumption decision in her role a firm manager.

[6] When $f'' = 0$, the second order condition for profit maximization is not satisfied; so that the size of the firm is indeterminate. For this case we draw this supply curve as a horizontal line at the breakeven price.

A numerical example

We illustrate this result with a specific example. Consider an economy with only one agent, Robinson, and two goods, leisure x_1 and burritos x_2. Burritos are produced from labor according to production function $y_2=|y_1|$, where leisure sold as labor input $y_1 = x_1 - \omega_1$. Robinson owns the burrito firm and is endowed with labor power. His utility and endowment are

$$U(x_1, x_2) = x_1 - \frac{(x_2-4)^2}{2} \text{ with } (\omega_1, \omega_2) = (4,0).$$

This economy's transformation function is $0 = 4 - x_1 - x_2$.

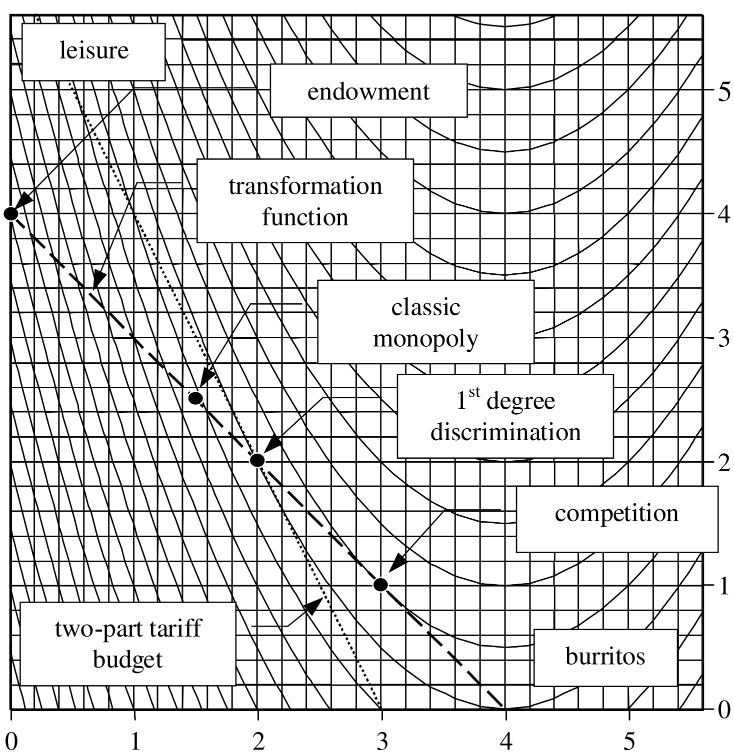
First consider a competitive market where Robinson is a price taker in both consumption and production decisions. Taking the price of leisure as the numeraire (p_1=1), the firm's profit is $\Pi=p_2y_2+y_1$. Substituting the production relation, and maximizing with respect to y_2 gives a flat supply curve at p_2=1.[7]

Substituting the budget constraint into utility, gives

$$U(x_1, x_2) = (4 - p_2x_2) - \frac{(x_2-4)^2}{2}.$$

Thus, the demand curve for burritos is $x_2 = 4 - p_2$. This linear demand curve is similar to those plotted in most textbook presentations of this topic. As shown in Figure 2, these are non-homogeneous preferences. This implies that the demand for leisure is $x_1 = m - 4p_2 + p_2^2$. In the competitive case the agent's income m is 4 because the firm breaks even. However, in the monopoly cases m can include profit and lump-sum payments, an effect that is overlooked in the partial equilibrium analysis. The competitive equilibrium occurs at $(x_1, x_2) = (1,3)$.

Figure 2. Competition is Pareto efficient, monopoly is not



Next consider the classic monopoly case

[7] This is not a proper solution. The second derivative of profit is zero, and there is no unique solution for y_2. The firm breaks even at any level of output.

$$\Pi = p_2 y_2 + y_1$$
$$\Pi = (4 - x_2)x_2 - x_2$$

Substituting for market clearing in the burrito market ($y_2 = x_2$) and the above transformation function. Profit is maximized at the monopoly equilibrium, $(x_1, x_2) = \left(\frac{5}{2}, \frac{3}{2}\right)$, with $p_2 = \frac{5}{2}$. The monopoly earns a profit of 9/4, which is added to Robinson's endowment for a total budget is 25/4; he spends 10/4 on leisure and 15/4 on burritos.

Now consider 1st-degree price discrimination. Now each consumer is offered a take-it-or-leave-it deal: (r, x_1, x_2) or $(0, 4, 0)$. The largest payment *r* is given by

$$U(x_1, x_2) - r = U(4, 0)$$

$$x_1 - \frac{(x_2 - 4)^2}{2} - r = 4 - \frac{(0 - 4)^2}{2}$$
$$r = 4 + x_1 - \frac{(x_2 - 4)^2}{2}.$$

The monopolist's profit is $\Pi = r + y_1$. Substituting for the payment, market clearing in the burrito market and the transformation function gives

$$\Pi = 8 - 2x_2 - \frac{(x_2 - 4)^2}{2}.$$

The first order condition is $x_2 = 2$, so that (r, x_1, x_2) is (4,2,2). The monopoly earns a profit of 2, which is added to Robinson's endowment for a total budget is 6; he spends 2 on leisure and pays 4 to the firms for the right to consume 2 burritos. Thus Robinson is on his budget constraint.

In the case of a two-part tariff, each consumer pays a flat fee for the right to enjoy burritos, plus a price for each burrito they consume. The fee, *r*, is the same as the *r* in first-degree price discrimination

$$r = 4 + x_1 - \frac{(x_2 - 4)^2}{2}.$$

But the profit function has changed. It is now $\Pi = r + p_2 x_2 + y_1$. Substituting for the payment, market clearing in the burrito market and the demand curve for burritos,

$$\Pi = 4 + (4 - x_2) - \frac{(x_2 - 4)^2}{2} + (4 - x_2)x_2 - x_2$$
$$\Pi = 8 + 2x_2 - \frac{(x_2 - 4)^2}{2} - x_2{}^2$$
$$\frac{\partial \Pi}{\partial x_2} = 0 = 2 - (x_2 - 4) - 2x_2$$

The first order condition is $x_2 = 2$, so that (r, x_1, x_2) is (4,2,2), with $p_2 = 2$ and a profit of 6.

The allocation is the same as in 1st-degree price discrimination. Profit is added to his endowment for a total budget is 10, out of which he spends 2 on leisure, 4 on the fee to the firms for the right to consume burritos, plus 4 on the per unit burrito charge. Robinson is again on his budget constraint (dotted line); see Figure 2. Price discrimination is inferior to perfect competition, although it is better than classic monopoly. It appears that the monopolist should recognize this and set production at the competitive equilibrium. This logical inconsistency may be resolved when this example is generalized to more than one agent, see Section 5.

4. A case of decreasing returns production

Now we consider an example of decreasing returns. Again the economy has only one agent, and two goods. As before burritos are produced from labor according to production function $y_2 = \sqrt{|y_1|}$. As before Robinson's utility and endowment are

$$U(x_1, x_2) = x_1 - \frac{(x_2 - 4)^2}{2} \text{ with } (\omega_1, \omega_2) = (4, 0).$$

First consider a competitive market where Robinson is a price taker in both consumption and production decisions. As before the firm's profit is $\Pi = p_2 y_2 + y_1$. Substituting the production relation, and maximizing with respect to y_2 gives the supply curve $p_2 = 2x_2$.

On the demand side, the demand curve for burritos is again $x_2 = 4 - p_2$, with the demand for leisure $x_1 = m - 4p_2 + p_2^2$. The competitive equilibrium occurs at $(x_1, x_2) = \left(\frac{20}{9}, \frac{4}{3}\right)$ with $p_2 = \frac{8}{3}$ and $\Pi = \frac{16}{9}$.

In the classic monopoly case

$$\Pi = p_2 y_2 + y_1$$
$$\Pi = (4 - x_2)x_2 - x_2^2$$

Substituting for market clearing in the burrito market ($y_2 = x_2$) and the transformation function, profit is maximized at $(x_1, x_2) = (3, 1)$ with $p_2 = 3$ and $\Pi = 2$.

Now consider 1st-degree price discrimination. The largest lump-sum payment r that the monopolist can demand is given by

$$U(x_1, x_2) - r = U(4, 0)$$

$$x_1 - \frac{(x_2 - 4)^2}{2} - r = 4 - \frac{(0 - 4)^2}{2}$$
$$r = 4 + x_1 - \frac{(x_2 - 4)^2}{2}.$$

Substituting this payment, market clearing in the burrito market and the transformation function into the monopolist's profit gives

$$\Pi = 8 - 2x_2^2 - \frac{(x_2 - 4)^2}{2}.$$

The 1st-degree equilibrium (r, x_1, x_2) is $\left(\frac{40}{25}, \frac{84}{25}, \frac{4}{5}\right)$. Our result is that not only does price discrimination differ from competition, but also it is less profitable than either classic monopoly or competition.

In the case of a two-part tariff the flat fee r is the same as in first-degree price discrimination, but the profit function has changed. Substituting for the payment, market clearing in the burrito market and the demand curve for burritos, the profit maximum is found from

$$\Pi = 4 + (4 - x_2^2) - \frac{(x_2 - 4)^2}{2} + (4 - x_2)x_2 - x_2^2$$
$$\frac{\partial \Pi}{\partial x_2} = 0 = 4 - (x_2 - 4) - 6x_2$$

The solution (r, x_1, x_2) is $\left(\frac{64}{49}, \frac{132}{49}, \frac{8}{7}\right)$ with $p_2 = \frac{20}{7}$ and $\Pi = \frac{224}{49}$.

Our diminishing returns case results differ significantly from the constant returns example. Here all regimes generate positive profits. The two-part tariff regime generates the greatest profits, while first-degree price discrimination generates the least, even less than competition. The equilibrium allocation differs in all four regimes. The economy is worst off under first-degree discrimination; classic monopoly is better; the two-part tariff is better yet; but only the competitive one is Pareto efficient.

5. Many agents

Two Agents

The conventional treatment of price discrimination does not address the distribution of monopoly ownership among the agents. This section shows that ownership inequality can adversely affect the efficiency of price discrimination equilibria.

Consider an identical economy, except that there are now two consumers, Friday and Robinson. Burritos are produced from labor according to production function $y_2=|y_1|$ as before, where leisure sold as labor input ($y_1 = x_{f1} + x_{r1} - \omega_{f1} - \omega_{r1}$). The transformation function for this economy is

$$0 = 4 - (x_{f1} + x_{r1}) - (x_{f2} + x_{r2}).$$

Individual utilities and endowments are identical,

$$U(x_{i1}, x_{i2}) = x_{i1} - \frac{(2x_{i2} - 4)^2}{4} \text{ with } (\omega_{i1}, \omega_{i2}) = (2,0), \text{ where } i=(f, r).$$

First we consider a competitive market. The firm's profit is $\Pi=p_2y_2+y_1$, taking the price of leisure as the numeraire (p_1=1).. Substituting the production relation, and maximizing with respect to y_2 gives the supply curve p_2=1. Substituting the budget constraint into utility, gives

$$U(x_{i1}, x_{i2}) = (m_i - p_2 x_{i2}) - \frac{(2x_{i2} - 4)^2}{4},$$

where $m_{i1} = \omega_{i1} + \Pi_i - r_i$ is the value of the consumer's endowment, plus his share of the firm's profit, minus any lump-sum payment. The demand curve for burritos is $x_{i2} = 2 - \frac{p_2}{2}$, so the market demand is $x_2 = 4 - p_2$. This implies that the demand for leisure is $x_{i1} = m_i - 2p_2 + \frac{p_2^2}{2}$. The competitive equilibrium occurs at $(x_{i1}, x_{i2}) = \left(\frac{1}{2}, \frac{3}{2}\right)$. Because profit is zero in this case, the result is the same regardless of ownership assumptions.

Next consider 1st-degree price discrimination. Now each consumer is offered a take-it-or-leave-it deal: (r_i, x_{i1}, x_{i2}) or $(0,2,0)$. The largest payment r_i is given by

$$U(x_{i1}, x_{i2}) - r_i = U(2,0)$$

$$x_{i1} - \frac{(2x_{i2} - 4)^2}{4} - r_i = 2 - \frac{(0-4)^2}{4}$$

$$r_i = 2 + x_{i1} - \frac{(2x_{i2} - 4)^2}{4}.$$

The firm's profit is $\Pi = r_f + r_r + y_1$. Substituting for the payments, market clearing in the burrito market ($y_2 = x_{r2} + x_{f2}$) and the transformation function gives

$$\Pi = 8 - 2(x_{f2} + x_{r2}) - \frac{(2x_{f2} - 4)^2}{4} - \frac{(2x_{r2} - 4)^2}{4}.$$

Taking partial derivatives, the first order condition is $x_{f2} = 1$ and $x_{f2} = 1$, or $x_2 = 2$ which is the same as in the one agent economy. So that (r_i, x_{i1}, x_{i2}) is (2,1,1) and profit is 2.

Now the equilibrium is affected by the distribution of ownership. At one extreme Robinson and Friday share ownership equally. Then profit income of 1 is added to each endowment for a total budget of 3, out of which each spends 1 on leisure and 2 on the payment to the monopoly. This is consistent with the solution above. At the other extreme, the equilibrium must be modified if one (Robinson) is the sole owner. Now Friday, who has only his endowment, chooses a corner solution at no leisure and 1 burrito (still better than his endowment). Robinson's budget is now 4; he chooses 2 leisure and 1 burrito. The consequence of inequality is that Friday does all the work and has no leisure, while Robinson does not work.

In the case of a two-part tariff, each consumer pays a flat fee to enjoy burritos, plus a price for each burrito they consume. The fee, r, is the same as the r in first-degree price discrimination

$$U(x_{i1}, x_{i2}) - r_i = U(2,0)$$
$$r_i = 2 + x_{i1} - \frac{(2x_{i2} - 4)^2}{4}.$$

Profit has changed; it is now $\Pi = r_r + r_f + p_2 x_2 + y_1$. Substituting for the payment, market clearing in the burrito market, the transformation function and the demand for burritos gives

$$\Pi = 4 + (4 - x_{f2} - x_{r2}) - \frac{(2x_{f2} - 4)^2}{4} - \frac{(2x_{r2} - 4)^2}{4} + (4 - (x_{f2} + x_{r2}))(x_{f2} + x_{r2}) - (x_{f2} + x_{r2}).$$

$$\Pi = 8 + 2(x_{f2} + x_{r2}) - \frac{(2x_{f2} - 4)^2}{4} - \frac{(2x_{r2} - 4)^2}{4} - (x_{f2} + x_{r2})^2$$

Taking partial derivatives,

$$\frac{\partial \Pi}{\partial x_{f2}} = 0 = 3 - 2x_{f2} - x_{r2}$$
$$\frac{\partial \Pi}{\partial x_{f2}} = 0 = 3 - 2x_{r2} - x_{f2}$$

whose solution is $x_{f2} = 1$ and $x_{r2} = 1$, or $x_2 = 2$. This gives in the same allocation as the one agent case; so that (r_i, x_{i1}, x_{i2}) is (2,1,1) with $p_2 = 2$ and profit is 6.

In the case that Robinson and Friday share the ownership equally, a profit of 3 is added to each endowment for a total budget of 5, out of which each spends 1 on leisure, 2 on the fee to the firm for the right to consume burritos, plus 2 on the per unit burrito charge; the budgets balance. This result is identical to the 1st-degree price discrimination case.

However, if Robinson is the sole owner, Friday chooses his endowment bundle: 2 leisure and no burritos. In the absence of sales to Friday, Robinson's profit drops to 3, so that his allocation is identical to the equal-shares economy. Since Friday chooses not to work, Robinson must provide all the labor for his burrito consumption.

Intermediate share distributions are more complex. As Robinson's share increases from equality, he works less, while still consuming one burrito. Moreover, Friday works more to supply Robinson until Friday hits the constraint of his leisure endowment. This point is reached when Robinson's ownership share rises to two thirds; at greater inequality Friday chooses his endowment. Robinson finds that he is better off with two-thirds share ownership than with sole ownership. Here is a case where greater equality implies greater efficiency.

There is a logical an inconsistency in this story. In setting the firm's pricing policy Robinson, the majority owner of the firm, may realize that he would be better off under competition $\left(U_r^{comp} = U\left(\frac{1}{2}, \frac{3}{2}\right) = \frac{1}{4}\right)$. This should happen when Robinson is the sole owner $\left(U_r^{sole} = U(1,1) = 0\right)$ and when he shares ownership equally $\left(U_r^{1/2} = U(1,1) = 0\right)$, however this inconsistency is gone when he owns two thirds $\left(U_r^{2/3} = U(2,1) = 1\right)$. When Robinson is the sole (or half) owner, he would be better off not collecting the lump-sum part of the tariff (keeping p_2 unchanged). In fact, there is a range of ownership shares that is inconsistent in this sense. When Robinson's share lies within (1/2, 13/24) or (2/3, 1), he is better off following the competitive rule than the two-part tariff.[8] Between these ranges are shares (13/24, 2/3,) for which Robinson prefers the two-part tariff.

Many Agents

[8] When Robinson's share is 13/24, then his profit income is 13/4. Thus his total budget is 21/4 and his consumption bundle is $\left(\frac{5}{4}, 1\right)$ yielding the same utility as the competitive regime.

Now extend this economy to n agents, but still one firm. Let the utility of the i^{th} consumer have by utility function

$$U_i(x_{i1},x_{i2}) = x_{i1} - \frac{(nx_{i2}-4)^2}{2n}\text{, with } (\omega_{i1},\omega_{i2}) = \left(\frac{4}{n},0\right);$$

the endowment of labor is identical. Burritos are produced from labor as before. The transformation function for this economy is $0 = 4 - \sum_{i=1}^{n} x_{i1} - \sum_{i=1}^{n} x_{i2}$. Initially each consumer owns an equal share of the single firm.

According to the same logic, the demand curve for burritos is $x_{i2} = \frac{4}{n} - \frac{p_2}{n}$ and the market demand is $x_2 = 4 - p_2$. Thus, the demand for leisure is $x_{i1} = m_i - 4p_2 + \frac{p_2^2}{n}$. And the competitive equilibrium occurs at $(x_{i1},x_{i2}) = \left(\frac{1}{n},\frac{3}{n}\right)$.

Consider 1st-degree price discrimination. Now each consumer is offered a take-it-or-leave-it deal: (r_i, x_{i1}, x_{i2}) or $\left(0,\frac{4}{n},0\right)$ such that

$$\begin{aligned}
U(x_{i1},x_{i2}) - r_i &= U\left(\frac{4}{n},0\right) \\
x_{i1} - \frac{(nx_{i2}-4)^2}{2n} - r_i &= \frac{4}{n} - \frac{(0-4)^2}{2n} \\
r_i &= \frac{4}{n} + x_{i1} - \frac{(nx_{i2}-4)^2}{2n}.
\end{aligned}$$

Thus the firm's profit is $\Pi = \sum_{i=1}^{n} r_i + y_1$. Substituting for the payments, market clearing in the burrito market and the transformation function gives

$$\Pi = \sum_{i=1}^{n}\left(\frac{4}{n} + x_{i1} - \frac{(nx_{i2}-4)^2}{2n}\right) + y_1$$

$$\Pi = 8 - \sum_{i=1}^{n}\left(2x_{i2} + \frac{(nx_{i2}-4)^2}{2n}\right)$$

Taking partial derivatives, the first order condition is $x_{i2} = \frac{2}{n}$ or $x_2 = 2$, so $(r_i, x_{i1}, x_{i2}) = \left(\frac{4}{n},\frac{2}{n},\frac{2}{n}\right)$ and profit is 2.

In the case of a two-part tariff, each consumer pays a flat fee to enjoy burritos, plus a price for each burrito they consume. The fee is the same as in first-degree price discrimination

$$\begin{aligned}
U(x_{i1},x_{i2}) - r_i &= U\left(\frac{4}{n},0\right) \\
r_i &= \frac{4}{n} + x_{i1} - \frac{(nx_{i2}-4)^2}{2n}.
\end{aligned}$$

Profit is now $\Pi = \sum_{i=1}^{n} r_i + p_2 x_2 + y_1$. Substituting for the payment, market clearing in the burrito market, the transformation function and the demand for burritos gives

$$\Pi = \sum_{i=1}^{n}\left(\frac{4}{n} + x_{i1} - \frac{(nx_{i2}-4)^2}{2n}\right) + p_2 x_2 + y_1.$$

$$\Pi = 8 + \sum_{i=1}^{n}\left(2x_{i2} - \frac{(nx_{i2}-4)^2}{2n}\right) - \left(\sum_{i=1}^{n} x_{i2}\right)^2$$

Taking partial derivatives,

$$\frac{\partial \Pi}{\partial x_{i2}} = 0 = 2 - nx_{i2} + 4 - 2\sum_{i=1}^{n} x_{i2}\ ,\ i{=}1\ldots n,$$

which can be written as a system of n equations,

$$\begin{bmatrix} n+2 & 2 & \cdots & 2 \\ 2 & n+2 & \cdots & 2 \\ \vdots & \vdots & \vdots & \vdots \\ 2 & 2 & \cdots & n+2 \end{bmatrix}\begin{bmatrix} x_{12} \\ x_{22} \\ \vdots \\ x_{n2} \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ \vdots \\ 6 \end{bmatrix}$$

The solution of this system is $x_{i2} = 2/n$ or $x_2 = 2$. This gives in the same allocation as the one agent case; so that $(r_i, x_{i1}, x_{i2}) = \left(\frac{4}{n}, \frac{2}{n}, \frac{2}{n}\right)$ with $p_2 = 2$ and profit is 6.

Here again ownership matters. Again there is a logical inconsistency when a single agent owns the firm (and all other agents choose their endowments in the two-part tariff regime): that agent would be better off setting the lump-sum tariff at zero. And again there is a range of ownership shares in which the majority owners prefer the two-part tariff rule to the competitive rule.

6. Conclusion

We conclude that the textbook analysis of perfect price discrimination needs to be rewritten. Our objection is that the conventional partial equilibrium argument for the efficiency of perfect price discrimination is flawed. Our counterexamples show that these conventional conclusions do not necessarily extend to general equilibrium. Consequently, two-part tariffs should not be advocated as a generally appropriate rule for utility regulation. We show that the textbook analysis is only valid as an unrealistic special case. The general approach is relevant not only to question of efficiency, but also to equity. Varying the monopoly ownership shares, we demonstrate that equity and efficiency can be intertwined; our example demonstrates that a reduction in equality can adversely affect efficiency.

7. References

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