

There are 72 points possible on this exam, 36 points each for Prof. Lozada's questions and Prof. Kiefer's questions. However, Prof. Lozada's questions are weighted differently from Prof. Kiefer's questions: Prof. Lozada's questions are worth 14 points, 14 points, and 8 points, while Prof. Kiefer's questions are worth 18 points, 9 points, and 9 points.

There are three sections on this exam:

- In the first section there are three questions; you should work all of them. The first is worth 14 points; the second is worth 14 points; and the last one is worth 18 points.
- In the second section there are two questions; you should work one of them. Each is worth 8 points.
- In the third section there are three questions; you should work two of them. Each is worth 9 points.

You have 4 hours and 30 minutes (that is, until 1:30 PM) to finish this test. This gives you about 45 minutes per question.

Do not use different colors in your answers because we grade looking at black-and-white photocopies of your exam.

It is helpful (but not required) if you put the number of the problem you are working on at the top of every page.

Good luck.

Section 1.

Answer all of the following three questions.

1. [14 points] Suppose a competitive firm produces output q and incurs total costs given by the following total cost function:

$$c(q) = \frac{1}{3}q^3 - q^2 + 2q + 1.$$

- (a) On one graph, sketch this firm's Marginal Cost curve, its Average Cost curve, and its Average Variable Cost curve.

Hint: "Average Variable Cost" means the average of: "total cost minus fixed cost," where "Fixed Cost" means $c(0)$.

Give numerical coordinates for:

- i. the minimum of the Marginal Cost Curve;
 - ii. the minimum of the Average Variable Cost Curve;
 - iii. the vertical-axis intercept of the Marginal Cost Curve, if it has such an intercept;
 - iv. the vertical-axis intercept of the Average Cost Curve, if it has such an intercept;
 - v. the vertical-axis intercept of the Average Variable Cost Cost Curve, if it has such an intercept.
- (b) Find the supply curve of this firm for *all non-negative* values of the price of q , which is denoted by " p ." Be sure to check second-order conditions.

Hint: if $ax^2 + bx + c = 0$ then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

2. [14 points] Consider a two-agent two-commodity pure exchange economy, with agents named Smith (" s ") and Jones (" j ") and commodities named apples (" a ") and bananas (" b "). Suppose the total number of apples in this economy is 1 and the total number of bananas in this economy is also 1, but both apples and bananas are completely divisible (which means they can be cut into arbitrarily small pieces).

Denote Smith's endowment by $\omega_s = (\omega_{sa}, \omega_{sb})$ and denote Jones's endowment by $\omega_j = (\omega_{ja}, \omega_{jb})$. Suppose Smith consumes x_{sa} apples and

x_{sb} bananas, and Jones consumes x_{ja} apples and x_{jb} bananas. Suppose Smith's utility function is $u_s = x_{sa}x_{sb}$, and suppose Jones's utility function is $u_j = x_{ja}x_{jb}$.

Suppose this economy has a social planner who wants the competitive equilibrium to be a certain arbitrary vector named

$$\bar{\mathbf{x}} = (\bar{x}_{sa}, \bar{x}_{sb}, \bar{x}_{ja}, \bar{x}_{jb}),$$

and in order to ensure that the competitive equilibrium does result in $\bar{\mathbf{x}}$, the social planner transfers T_a apples from Smith to Jones and transfers T_b bananas from Smith to Jones, all before any trading occurs. Note that T_a and T_b could take any sign (that is, they might not be positive). Also, note that because $\bar{\mathbf{x}}$ is assumed to be the outcome of a competitive equilibrium, it cannot be *completely* arbitrary, only somewhat arbitrary.

Find the values of T_a and T_b .

Hint: The values of T_a and T_b may not be unique.

3. [18 points]

Comcast and DirectTV produce TV services y_i ($i = c$ or d) in a duopolistic TV market. They have identical cost functions

$$c_i = c(w_1, w_2, y_i) = y_i \sqrt{w_1 w_2},$$

where w_1 is the rental rate for capital x_1 and w_2 is the wage rate for labor x_2 .

- (a) What is the Comcast production function? Is it homogeneous of degree 1? Monotonic? Convex? Plot the isocost and isoquant curves; give an interpretation these curves.
- (b) Suppose that the rental rate for capital is $w_1 = 2$; the wage rate is $w_2 = 2$, and that both markets are competitive. The direct demand functions are

$$\begin{aligned} \text{Comcast} \quad y_c &= 4 - p_c + \frac{1}{2}p_d \\ \text{DirectTV} \quad y_d &= 4 - p_d + \frac{1}{2}p_c. \end{aligned}$$

What is the equilibrium, expressed as (y_c, y_d, p_c, p_d) ? What is Comcast's netput vector, $(y_c, -x_{1c}, -x_{2c})$?

- (c) What is the outcome in the case that they come to a differentiated-product-Bertrand equilibrium? Plot the best response curves to illustrate. Why is it a Nash equilibrium?
- (d) Now suppose that Comcasts workers unionize and succeed in raising their wage to $w_2 = 8$, everything else the same (including nonunion DirectTV). What is your intuition about the effect of this event on this market? Find the new differentiated-product-Bertrand equilibrium. Comment.
- (e) Returning to $w_2 = 2$ for Comcast, now suppose that the two firms form a cartel, but continue to market both brands of TV service. What is the equilibrium?
- (f) Assuming that the cartel and Bertrand equilibriums are the only possibilities, fill in the payoff matrix.

profit payoffs: (Comcast, DirectTV)		DirectTV	
		cartel	Bertrand
Comcast	cartel		
	Bertrand		

- (g) Suppose that this game is repeated infinitely many times. Consider the *mutual punishment* threat:
- play cartel in the first game; thereafter play cartel,
 - unless the either rival plays Bertrand in any previous game, then play Bertrand.

What outcome will occur if both players follow this strategy? Under what circumstances is the (cartel, cartel) outcome a subgame-perfect Nash equilibrium of the infinitely repeated game?

Section 2.

Answer one of the following two questions.

1. [8 points] Suppose a competitive firm uses water w to produce output q according to the production function $q = \sqrt{w}$. Suppose the price of water is 1.
 - (a) The firm's Board of Directors is considering hiring a new Chief Executive Officer ("CEO") who claims to be able to change the firm's production function to $q = 2\sqrt{w}$. How much should the firm be willing to pay this new CEO?
 - (b) The notion of a firm having *two* production functions—namely $q = \sqrt{w}$ under the old CEO and $q = 2\sqrt{w}$ under the new CEO—is completely inconsistent with the standard neoclassical idea of “a production function.”
 - i. Describe this situation in a way that is compatible with standard neoclassical idea of “a production function.”
 - ii. Once you have formulated the correct production function, graph it. This may require a three-dimensional graph, but it should be a rather easy one to draw.
 - iii. Is the technology monotonic? convex? regular? Why?
 - iv. Are all inputs to production being paid their marginal product? Why or why not? (If you think there are good arguments on both sides of this question, you need not decide which arguments are better: just give both sides' arguments.)

Note that in the US, the position of CEO is almost always a full-time position held by one person. Assume that is true in this problem.

2. [8 points]
 - (a) A monopolist produces output “ x ,” faces a demand curve $p(x, \alpha)$, and has a cost function $c(x)$. The parameter “ α ” shifts consumers' demand for his product up, and the monopolist can choose α , but the cost to the monopolist of α is 5α .
Implicitly find the monopolist's optimal choices.

(b) If, in part (a),

$$p(x, \alpha) = 10\alpha - 3x \quad \text{and}$$

$$c(x) = x^2,$$

are the second-order conditions satisfied when the first-order conditions are?

Section 3.

Answer two of the following three questions.

1. [9 points]

Consider a duopoly strategy game with three options: labeled *left*, *middle* and *right*. The profit payoff matrix is

profit payoffs: (Airbus, Boeing)		Boeing		
		<i>left</i>	<i>middle</i>	<i>right</i>
Airbus	<i>left</i>	12, 12	5, 14	1, -1
	<i>middle</i>	14, 5	7, 7	2, 0
	<i>right</i>	-1, 1	0, 2	3, 3

- (a) Explain why the *middle* dominates the *left* for both players. Are there any Nash equilibriums in a one-shot, simultaneous game? Explain.
- (b) Two games are played; each is simultaneous. Consider the strategic threat:
- play *left* in the first game,
 - if rival plays *left* in the first, then play *middle* in second,
 - if rival does not play *left* in the first, then play *right* in second

Under what circumstances is [1st game: (*left*, *left*), 2nd game: (*middle*, *middle*)] a Nash equilibrium?

- (c) Is punishment for deviation [1st: (*left*, *middle*), 2nd: (*right*, *right*)] subgame perfect?

2. [9 points]

The postulate of *methodological individualism* underlies all public choice analysis. In trying to explain governmental actions, we begin by analyzing the behavior of the individuals who make up the government. In a democracy these are the voters, their elected representatives, and appointed bureaucrats. The postulate of methodological individualism has a normative analogue. The actions of government ought to correspond, in some fundamental way, to the preferences of the individuals who these actions effect, the citizens of the state. The postulate of normative individualism underlies much of normative analysis in public choice.

—Dennis Mueller

Explain the term methodological individualism. Discuss its role in neo-classical microeconomics. Discuss and evaluate the alternative schools of thought concerning methodology.

3. [9 points]

Tarzan, Jane and their pet chimpanzee Cheetah live in the jungle. They live off bananas that do not grow naturally in their particular jungle; they must be cultivated. Tarzan and Jane each own personal banana groves. But Cheetah is incapable of farming and totally dependent on their generosity for her sustenance.

Believing in the principle that “all primates are created equal,” Tarzan and Jane wish to see that Cheetah is well fed. They have identical utility functions and endowments:

$$\begin{array}{lll} \text{Tarzan} & U_t = x_t G & \omega_t = 3, \\ \text{Jane} & U_j = x_j G & \omega_j = 3 \end{array}$$

where ω stands for endowment (in bunches of bananas/day) of each human, x_i is personal consumption and G is Cheetah’s consumption. The jungle society’s transformation function is given by

$$T(G, x_t, x_j) = x_j + x_t + G - \omega_j - \omega_t.$$

- (a) Plot best response curves in contribution space (g_t — g_j); contributions are defined as $g(i) = \omega(i) - x(i)$. Find the Nash equilibrium; express your answer as (G, x_j, x_t) .
- (b) Find all Pareto efficient allocations. Illustrate your answer in both g_t — g_j space and U_t — U_j space.
- (c) Suppose Tarzan and Jane agree that the social welfare is defined by

$$W = \sqrt{U_j} + \sqrt{U_t}.$$

Find the social optimum.