

## MACROECONOMICS QUALIFYING EXAM

**Answer all questions:**

*Question 1.* Consider the *IS-PC-MR* model,

$$\begin{aligned} x_t &= -(r_{t-1} - r_S), & IS \\ \pi_t &= \pi_{t-1} + x_t, & PC \\ \pi_t - \pi^T &= -x_t, & MR \end{aligned}$$

where the endogenous variables are the inflation rate  $\pi_t$ , the GDP gap  $x_t$  and the lagged real interest rate  $r_{t-1}$ . The exogenous variables are the lagged inflation rate  $\pi_{t-1}$ , the stabilizing real interest rate  $r_S$  and the inflation target  $\pi^T$ .

- (a) Explain the logic of these three equations; illustrate your answer diagrammatically.
- (b) What is the long-run equilibrium of this model?
- (c) Specifically for the Phillips curve, motivate the general form with both perfect and imperfect goods and labor markets and critically examine its validity. Discuss implications for demand management. Discuss whether this model incorporates rational expectations. (Use symbols and math if it helps you to make your point. It is, however, not necessary.)
- (d) Obtain the total differential of these equations. Write these differentials as a matrix equation,  $Jdy=dx$ , where  $J$  is the Jacobian matrix,  $dy$  a vector of endogenous partials and  $dx$  a vector of exogenous effects. What condition is necessary to insure that equilibrium exists? Is it satisfied?
- (e) Evaluate the sign of  $\frac{\partial x_t}{\partial \pi_{t-1}}$ . Illustrate the effect of an increase in the lagged inflation rate diagrammatically and interpret.
- (f) Evaluate the sign of  $\frac{\partial \pi_t}{\partial \pi^T}$ . Illustrate the effect of an increase in the inflation target diagrammatically and interpret.

*Question 2.* The system of differential equations of the Kaldor model is

$$\begin{aligned} \dot{Y} &= \alpha(I[Y, K] - S[Y, K]) \\ \dot{K} &= I[Y, K] - \delta K \end{aligned}$$

Assume  $0 < \alpha, \delta < 1$ ; and that savings increase linearly and investment non-linearly with demand, so that the Keynesian stability condition for the goods market is violated at normal levels of demand, but satisfied for low and high levels of demand.

- (a) Briefly motivate both differential equations.
- (b) Derive Jacobian, isoclines, and present the phase diagram with arrows of motion. State further assumptions as necessary, and briefly explain.

*Question 3.* Let the growth rate of capital be  $g$ , and the valuation ratio  $q = \frac{P_E K}{PK}$ , where  $P_E K$  is the market value of issued equity and  $PK$  the replacement cost of physical assets. Suppose that  $J$  below summarizes dynamic feedback of a linearized model in  $g$  and  $q$ .

$$J = \begin{bmatrix} \frac{\partial \dot{g}}{\partial g} < 0 & \frac{\partial \dot{g}}{\partial q} > 0 \\ \frac{\partial \dot{q}}{\partial g} < 0 & \frac{\partial \dot{q}}{\partial q} > 0 \end{bmatrix}$$

Briefly motivate the assumed sign pattern. Discuss stability, show a phase diagram; briefly explain.

*Question 4.* Neoclassical  $q$ -theory:

Assume a representative firm has a quadratic profit function

$$\pi(K) = K - K^2,$$

and faces positive and increasing adjustment costs

$$C(K) = I^2.$$

Further, the purchase price of the physical asset  $K$  is constant at  $P=1$ , the discount rate is  $\rho=0.1$ , and  $\delta=0$ . Set up the optimization problem, and analyze the model. Present the Jacobian matrix; discuss sign pattern, the trace and determinant. Derive isoclines, and draw phase diagram with arrows of motion. Briefly explain.

**Answer two questions:**

*Question 5.*

Consider a simple economy of  $N$  identical worker-investor-consumers. Each is endowed with 1 unit of labor power and nominal money  $m_o$ . Workers offer their labor to a single firm independently of the wage rate. The single firm produces a consumption good  $Y$  according to the production function

$$Y = \sqrt{N},$$

Taking the wage rate and price as given, the firm maximizes profit. In their role as investors, all workers receive equal shares of the firm's profits,

$$\Pi = pY - wN,$$

where  $p$  is the price of the consumption good produced and  $w$  is the nominal wage.

In their role as consumers, everybody allocates their budgets between consumption  $c_i$  and real money balances according to the utility function

$$U_i = \sqrt{c_i \left( \frac{m_i}{p} \right)},$$

where  $m_i$  is the nominal money balance held by the  $i^{\text{th}}$  consumer. Consumers face the budget constraint

$$(1 - \lambda) \left( w + \frac{\Pi}{N} \right) + m_o = pc_i + m_i,$$

where  $\lambda$  is the income tax rate. The consumption decision is made according to competitive assumptions; that is, consumers take  $w$ ,  $p$ ,  $\lambda$  and  $\Pi$  as given.

According to the constitution of this society, there is no government spending, although income taxation is permitted. Equilibrium in the goods market is described by

$$Y = C = \sum_{i=1}^N c_i.$$

The government controls the supply of nominal money so that equilibrium in the money market is given by

$$M = \sum_{i=1}^N m_i.$$

Assume that  $p$ ,  $w$ ,  $Y$ ,  $C$  and  $\lambda$  are endogenous, while  $M$ ,  $m_o$  and  $N$  are exogenous.

- Derive the aggregate consumption function for this microeconomic description. What is the implied *marginal propensity to consume*? Compare this model to the Quantity Theory of Money. What is the implied *velocity of money*?
- Show that this model implies a particular budget constraint for the government. If the money supply  $M$  is exogenous, then the tax rate  $\lambda$  must be endogenous. Explain why.
- Given that  $M=10$ ,  $m_o=1/10$ , and  $N=100$ , find equilibrium values for  $Y$ ,  $p$  and  $w$ . Illustrate your answer.

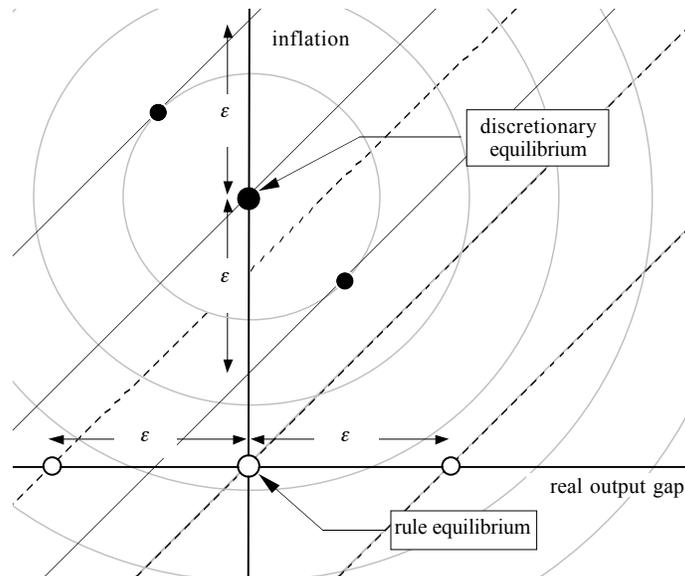
Question 6. Consider a simple macroeconomy:

$$\begin{aligned} \frac{w}{p} &= F_N(N), & \text{labor demand,} & & F_{NN} < 0, \\ Y &= F(N), & \text{production function,} & & F_N > 0, \\ \frac{M}{p} &= m(Y), & \text{money market equilibrium,} & & m_Y > 0, \end{aligned}$$

where  $\frac{w}{p}$ ,  $Y$  and  $p$  are endogenous; and  $N$  and  $M$  are exogenous. All variables are labeled as in Sargent.

- Obtain the total differential of these equations. Write these differentials as a matrix equation,  $Jdy=dx$ , where  $J$  is the Jacobian matrix,  $dy$  a vector of endogenous partials and  $dx$  a vector of exogenous effects. What condition is necessary to insure that equilibrium exists? Is it satisfied?
- Use the implicit function theorem to evaluate the signs of  $\frac{\partial Y}{\partial N}$  and  $\frac{\partial \left(\frac{w}{p}\right)}{\partial N}$ . Interpret your results. Illustrate your answer with a diagram.
- Discuss whether this is a classical or Keynesian model. Does it exhibit neutrality? Dichotomy?

Question 7. The diagram below depicts the theories associated with a debate between two schools of thought: one advocates activist discretionary policy, while the other advocates nondiscretionary policy rules. It is partially labeled; you should add additional explanation and include it with your answer.



- Explain the model that underlies each of the two equilibria labeled on the diagram. Which variables are endogenous in each model, which are exogenous?
- What assumption does each model make about the formation of expectations?
- According to this diagram, which school of thought is more effective with respect to macrostabilization?