Spring 2004 Microeconomics Qualifying Exam

There are three sections on this exam:

- In the first section there are two questions; you should work both of them.
- In the second section there are three questions; you should work two of them.
- In the third section there are three questions; you should work two of them.

You have 4 hours and 30 minutes (that is, until 1:30 PM) to finish this test. This gives you about 45 minutes per question.

Good luck.

Section 1.
Answer both of the following two questions.

1. Suppose there are two consumers, Tom and Harry, and each obtains utility from hours of sleep, s, and from consuming good x. Tom and Harry's utility functions are

$$u_t(s_t, x_t) = s_t\, x_t \quad \text{and} \quad u_h(s_h, x_h) = s_h\, x_h$$

respectively. Neither Tom nor Harry has any initial endowment of x; their initial endowments of time are

$$\omega_t = 1 \quad \text{and} \quad \omega_h = 2\,,$$

respectively, and they divide their time into hours of sleep and hours of work "w".

The total amount of good x in the economy is created from the total amount of work w in the economy according to the production function $x = 4w$.

(a) Find the Pareto Optimal allocations of s_t, x_t, s_t, and s_h. Hint: the answer can be written in several forms; one is

$$\begin{aligned} s_t &= \tfrac{1}{2}\alpha & x_t &= 2\alpha \\ s_h &= \tfrac{3}{2} - \tfrac{1}{2}\alpha & x_h &= 6 - 2\alpha \end{aligned}$$

where α is some constant.

(b) Suppose Tom and Harry form a two-person competitive economy. However, suppose there is government interference in this economy: the government subsidizes Tom's purchases of x and taxes Harry's purchases of x, using so-called *ad valorem* ["according to value"] taxes τ. (Note that Tom is poor and Harry is rich because of the differences in their endowments. Also, ignore any constraint concerning the government's budget balance.)

i. Explain why Tom solves the problem

$$\max_{s_t, x_t} s_t\, x_t \quad \text{s.t. } (1 - s_t)\, p_w = p_x\, x_t\, (1 - \tau)$$

and why Harry solves the problem

$$\max_{s_h, x_h} s_h\, x_h \quad \text{s.t. } (2 - s_h)\, p_w = p_x\, x_h\, (1 + \tau)\,.$$

ii. Show that in competitive equilibrium,

$$s_t = \tfrac{1}{2} \quad \text{and} \quad s_h = 1\,.$$

iii. Either show that in competitive equilibrium

$$x_t^* = \frac{6\,(1+\tau)}{3-\tau}$$

or show that in competitive equilibrium

$$x_h^* = \frac{12\,(1-\tau)}{3-\tau}$$

(you do not have to explain both of these).

iv. From your answers to (a), (b-ii), and (b-iii), explain that the allocation in the two-person competitive economy with distortionary taxes can only be Pareto Optimal if the tax rate τ is zero.

2. Mac produces burgers y by combining grills x_1 and labor x_2.

(a) Mac's cost function is

$$c(w_1, w_2, y) = (w_1 + w_2)y\,,$$

where w_1 is the rental rate for grills and w_2 the wage rate. What is her production function? Is it homogeneous of degree 1? Monotonic? Convex? Plot the isocost and isoquant curves; give an interpretation of the slopes for this case.

(b) Suppose that the rental rate for grills is $w_1 = 1$; the wage rate is $w_2 = 1$, and that demand is given by the function

$$p(y) = 3 - y\,.$$

And, suppose that this market is competitive. What is its equilibrium?

(c) Now suppose that Mac is a monopolist. What is the equilibrium?

(d) Now suppose two firms (Wendy and Mac) share this market; they behave as a Cournot duopoly. They have identical costs. What is the equilibrium? Why is it a Nash equilibrium?

(e) Construct a welfare analysis of the monopoly and Cournot equilibria, as compared with competition. Illustrate your answer. Calculate the deadweight losses in each case.

(f) Fill in the empty cells in the following matrix of profit payoffs.

		Mac: cartel	Mac: Cournot
Wendy	cartel	$(\frac{1}{8}, \frac{1}{8})$	$(\frac{5}{36}, \frac{5}{48})$
	Cournot		

(g) Under what circumstances is the (cartel, cartel) outcome a subgame-perfect Nash equilibrium of an infinitely repeated game?

(h) What are the implications of this example for public policy?

Section 2.
Answer two of the following three questions.

1. Attached to this exam is an excerpt from pages 167 and 168 of Varian's textbook. This excerpt ends with Varian stating that

$$p^0 > p^1 \Longrightarrow EV > CV\,.$$

   (a) If $p^0 < p^1$, is EV greater than or less than CV?

   (b) Related to compensating and equivalent variation are:

   - "willingness to pay" an amount of money in order to avoid suffering the price increase from p^0 to p^1; and
   - "willingness to accept" an amount of money in order to accept the price increase from p^0 to p^1.

   Is "willingness to pay" equal to CV or to EV? Is "willingness to accept" equal to CV or to EV? Why?

2. Consider a cost-minimizing firm which produces an output y using two inputs x_1 and x_2 according to the production function $y = f(x_1, x_2)$. Make the usual assumptions that the firm takes the prices of x_1 and x_2 to be fixed, that both $\partial f/\partial x_1$ and $\partial f/\partial x_2$ are strictly positive, and that there are diminishing returns to each input.

   Call an input "inferior" if when output increases, the firm chooses to use less of this input.

   (a) Is it possible for both x_1 and x_2 to be inferior (simultaneously)? You should be able to answer this without solving any optimization problem; indeed, undergraduate students who do not know calculus should be able to answer this.

   (b) Under what conditions is x_1 inferior? (This is not a question undergraduates could solve.)

   (c) Under what conditions is x_2 inferior?

   (d) Use the answers to (a), (b), and (c) to argue that it is impossible for f''_{12} to be very negative.

   (e) Could x_1 or x_2 ever actually be inferior? When?

3. Baby Alice has two grandfathers, Grandpa Smith and Grandpa Jones. Suppose there is one good, called "x", and suppose the consumption

of that good by Baby Alice, Grandpa Smith, and Grandpa Jones is x_A, x_S, and x_J, respectively.

Grandpa Smith has an endowment of 3 units of good x, of which he consumes x_S and gives x_{SA} to Baby Alice:

$$x_S + x_{SA} = 3\,.$$

Grandpa Jones has an endowment of 3 units of good x, of which he consumes x_J and gives x_{JA} to Baby Alice:

$$x_J + x_{JA} = 3\,.$$

Baby Alice has no endowment of good x; all of her consumption consists of gifts from her grandfathers:

$$x_A = x_{SA} + x_{JA}\,.$$

Baby Alice's utility function is

$$u_A(x_A) = \ln x_A\,.$$

Grandpa Smith's utility function is

$$u_S(x_S, u_A) = \ln x_S + u_A\,.$$

Grandpa Jones's utility function is

$$u_J(x_J, u_A) = \ln x_J + u_A\,.$$

(So the grandfathers care about their own consumption and the utility of Baby Alice.)

(a) Find the values of x_{SA}, x_{JA}, x_A, x_S, x_J, u_S, u_J, and u_A. Assume that each grandfather takes the other grandfather's actions as fixed. (So this is technically an equilibrium in the sense of Nash.)

(b) Denote the values found in part (a) as x^*_{SA}, x^*_{JA}, x^*_A, x^*_S, x^*_J, u^*_S, u^*_J, and u^*_A. Now suppose that instead of giving x^*_{SA} and x^*_{JA}, each grandfather increases his gift by $\epsilon > 0$. Show that for sufficiently small ϵ, this change increases *everyone's* utility compared with the situation of part (a).

Section 3.

Answer two of the following three questions.

1. Consider the duopolistic Thai beer market. The direct demand functions are

$$\begin{aligned} \textit{Singha:} \quad & q_s = 30 - p_s + \tfrac{1}{3}p_h \\ \textit{Heineken:} \quad & q_h = 30 - p_h + \tfrac{1}{3}p_s\,. \end{aligned}$$

These are differentiated products. The marginal and average costs of production are 20 Baht/bottle for both firms.

   (a) Find the Bertrand equilibrium; express your answer as (p_s, p_h). Illustrate your answer by plotting reaction curves.

   (b) Why is this also a Nash equilibrium?

   (c) Suppose that the government imposes a tax of t per bottle on Heineken only. Find $\partial p_s/\partial t$. Illustrate your answer.

2. Tom, Dick and Harry share an apartment at Quasilinear Gardens. They each derive benefit from apartment cleanliness G, a public good, and leisure x, a private good. Production of cleanliness and leisure takes place according to the transformation function,

$$T(G, x_t, x_d, x_h) = G + x_t + x_d + x_h - \omega_t - \omega_d - \omega_h = 0.$$

Their utility functions and endowments are

$$\begin{aligned} &\text{Tom,} \quad && U_t = x_t + 4\ln(G), \quad && \omega_t = 10, \\ &\text{Dick,} \quad && U_d = x_d + 6\ln(G), \quad && \omega_d = 10, \\ &\text{Harry,} \quad && U_h = x_h + 8\ln(G), \quad && \omega_h = 10. \end{aligned}$$

   (a) Find the Nash equilibrium; express your answer as (G, x_t, x_d, x_h).

   (b) Find the Lindahl equilibrium.

   (c) Find the Bowen equilibrium.

   (d) Compare all three equilibria, discussing the extent to which each is "ideal."

3. A democratic society consists of many citizens, identical except for their employment status. There are only two time periods: the present

($t = 1$) and the future ($t = 2$). Each individual has the following utility function,

$$U = E\left(\sqrt{c_1^j} + \sqrt{c_2^j}\right) \quad \text{with} \quad j \in \{\text{employed, unemployed}\}$$

where c_t^j is consumption in the t^{th} period. *Note: they do not discount future utility.*

The probability that an employed in period 1 will lose her job for period 2 is $\phi = 0.04$ (the firing rate), while the probability that an unemployed will gain a job is $\nu = 0.76$ (the hiring rate). The unemployment rate is 0.05 in period 1.

At the beginning of period 1 an election sets a tax τ on the employed to finance an unemployment insurance program for both periods. Employed consumption is $c_t^e = 1$ before taxes, and unemployed consumption is $c_t^u = 0$; afterward, $c_t^e = 1 - \tau$ and $c_t^u = f_t$. On election day voters know their employment status in period 1, but not in period 2. Total tax collections equal benefits paid in each period, so that the government budget always balances.

(a) Explain the nature of uncertainty and risk aversion in this problem.

(b) What tax does the employed majority prefer? What is the implied benefit level in period 2?

(c) Discuss the wider implications of this model for studying social conflict.

Varian p.167
For Section 2 Problem 1

10.5 Consumer's surplus as an approximation

We have seen that consumer's surplus is an exact measure of the compensating and equivalent variation only when the utility function is quasilinear. However, it may be a reasonable approximation in more general circumstances.

For example, consider a situation where only the price of good 1 changes from p^0 to p' and income is fixed at $m = m^0 = m'$. In this case, we can use the equation (10.1) and the fact that $\mu(\mathbf{p}; \mathbf{p}, m) \equiv m$ to write

$$EV = \mu(p^0; p', m) - \mu(p^0; p^0, m) = \mu(p^0; p', m) - \mu(p'; p', m)$$
$$CV = \mu(p'; p', m) - \mu(p'; p^0, m) = \mu(p^0; p^0, m) - \mu(p'; p^0, m).$$

We have written these expressions as a function of p alone, since all other prices are assumed to be fixed. Letting $u^0 = v(p^0, m)$ and $u' = v(p', m)$ and using the definition of the money metric utility function given in Chapter 7, page 109, we have

$$EV = e(p^0, u') - e(p', u')$$
$$CV = e(p^0, u^0) - e(p', u^0).$$

Finally, using the fact that the Hicksian demand function is the derivative of the expenditure function, so that $h(p, u) \equiv \partial e / \partial p$, we can write these expressions as

$$EV = e(p^0, u') - e(p', u') = \int_{p'}^{p^0} h(p, u')\, dp$$
$$CV = e(p^0, u^0) - e(p', u^0) = \int_{p'}^{p^0} h(p, u^0)\, dp. \tag{10.2}$$

It follows from these expressions that the compensating variation is the integral of the *Hicksian* demand curve associated with the initial level of

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utility, and the equivalent variation is the integral of the Hicksian demand curve associated with the final level of utility. The correct measure of welfare *is* an integral of a demand curve—but you have to use the Hicksian demand curve rather than the Marshallian demand curve.

However, we can use (10.2) to derive a useful bound. The Slutsky equation tells us that

$$\frac{\partial h(p,u)}{\partial p} = \frac{\partial x(p,m)}{\partial p} + \frac{\partial x(p,m)}{\partial m} x(p,m).$$

If the good in question is a normal good, the derivative of the Hicksian demand curve will be larger than the derivative of the Marshallian demand curve, as depicted in Figure 10.2.

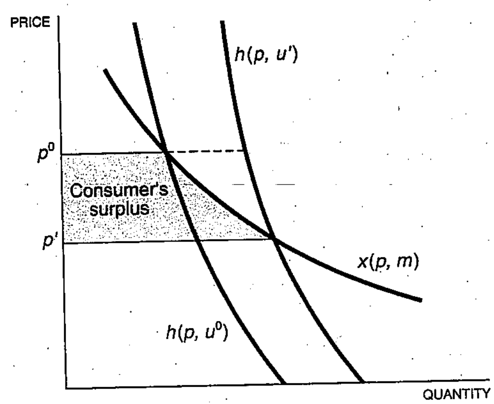

Figure 10.2 **Bounds on consumer's surplus.** For a normal good, the Hicksian demand curves are steeper than the Marshallian demand curve. Hence, the area to the left of the Marshallian demand curve is bounded by the areas under the Hicksian demand curves.

It follows that the area to the left of the Hicksian demand curves will bound the area to the left of the Marshallian demand curve. In the case depicted, $p^0 > p^1$ so all of the areas are positive. It follows that $EV >$ consumer's surplus $> CV$.