

7008 QUALIFYING EXAM 2003

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This is the qualifying exam in macro by Hans G. Ehrbar. Answer the following four questions. You have choices for each question.

- As your first question please do either question 1 about the aggregation problem, or question 3 about rational expectations in an econometric model.
- The second question is either a simple IS-LM model 2 or a simple maximization problem 6.
- The third question is an essay question: either 7 or 8.
- The fourth question goes through one of the models discussed in class in some detail. Either 4 or 5.

Problem 1. *This exercise, derived from [Sar87, pages 7, 8, 9], shows how an aggregate production function can come about as the aggregation of many individual production functions.*

Assume the economy has n firms, which produce the same good and sell it at the same price p . The i th firm, which has inherited from the past the amount of capital K_i , hires that amount of labor N_i which maximizes its profit. All firms have identical production functions $Y_i = F(K_i, N_i)$ which exhibit constant returns to scale, i.e., which satisfy $F(\lambda K, \lambda N) = \lambda F(K, N)$, with $F_2 > 0$, and $F_{22} < 0$. All firms sell their product at the same price p and hire labor at the same wage w .

- a. (3 points) Show that in this case K_i/N_i is the same for all i (call it $K_i/N_i = k$).
- b. (2 points) Show furthermore that the real wage satisfies $\frac{w}{p} = F_2(K, N)$, where $K = \sum_i K_i$ and $N = \sum_i N_i$.
- c. (4 points) Finally show that the total product of the economy is $Y = F(K, N)$, where $Y = \sum Y_i$.
- d. (4 points) The production function $Y = F(K, N)$ is constant returns to scale, i.e., $F(\lambda K, \lambda N) = \lambda F(K, N)$ for all $\lambda > 0$. Does this imply that $F(\sum K_i, \sum L_i) = \sum F(K_i, L_i)$? If yes, give a proof; if no, give a counterexample.

Problem 2. *In the usual Keynesian ISLM models the money supply is assumed to be exogenous. If one models the money supply more carefully, one sees that the money supply increases with the interest rate, since banks will want to keep their excess reserves low when interest rates are higher. Does this increase or decrease the Keynesian government expenditure multiplier? You may either give an intuitive explanation, or use a simple ISLM model in which money supply depends on the interest rate, and compute the multiplier.*

Problem 3. *This question follows the original article [SW76] much more closely than [HvdP02] does. Sargent and Wallace first reproduce the usual argument why "activist" policy rules, in which the Fed "looks at many things" and "leans against the wind," are superior to policy rules without feedback as promoted by the monetarists.*

They work with a very stylized model in which national income is represented by the following time series:

$$(1) \quad y_t = \alpha + \lambda y_{t-1} + \beta m_t + u_t$$

- a* technological constant indicating decreasing returns to scale in production
g government expenditures in real terms
l length of workday (the same for all employed workers)
*m*₀ initial wealth plus nonlabor income of every consumer
*m*_{*i*} money holding at end of period by consumer *i*
N number of workers/consumers
p price of the good
u rate of unemployment
w wage rate
*x*_{*i*} consumption of consumer *i*
y total output
z labor demand in labor hours. Note that $z = N(1 - u)l$.

The workers/consumers have utility function $U_i = x_i^2 \frac{m_i}{p}$, government has government expenditures *g*, and the technology is such that for producing output *y* one needs the amount *z* of labor, where

$$(7) \quad z = y\left(1 + \frac{ay}{2}\right).$$

(An alternative description of the technology would be the production function $y = (\sqrt{1 + 2az} - 1)/a$.)

- a. (4 points) Show that the notional demand for labor *z* and the notional supply of goods *y* are

$$(8) \quad z = \frac{1}{2a} \left(\frac{p^2}{w^2} - 1 \right)$$

$$(9) \quad y = \frac{1}{a} \left(\frac{p}{w} - 1 \right).$$

- b. (4 points) Show that the goods and money demand of an employed worker *i* is

$$(10) \quad x_i = \frac{2}{3p} (m_0 + wl)$$

$$(11) \quad m_i = \frac{1}{3} (m_0 + wl)$$

and the goods and money demand of an unemployed worker *j* is

$$(12) \quad x_j = \frac{2}{3p} m_0$$

$$(13) \quad m_j = \frac{1}{3} m_0.$$

These workers take their labor hours as given and choose consumption and money balances which maximize their utility.

- c. (2 points) Show that aggregate demand (total consumption plus government expenditures) when the rate of unemployment is *u*, and neither employed nor unemployed workers are constrained in consumption, is, in real terms,

$$(14) \quad y = g + \frac{2N}{3p} (m_0 + w(1 - u)l).$$

- d. (3 points) Assume the firm hires the number of workers needed to produce a given output *y*, which is below full employment output. Show that the rate of unemployment *u* satisfies

$$(15) \quad 1 - u = \frac{y}{Nl} \left(1 + \frac{ay}{2} \right).$$

- b. (3 points) Firms are small; each firm has one job only which is vacant when the firm enters the market. Vacant firms engage in hiring, which costs γ_0 per time unit. If the job is filled, the firm produces. Its output is 1 product per time unit. The firm is able to sell this product instantly at price p . If r is the interest rate and w the expected value of the wage paid by the firm, this gives the two arbitrage equations

$$(24) \quad rJ_V dt = -\gamma_0 dt + q(\theta) dt [J_O - J_V]$$

$$(25) \quad rJ_O dt = (p - w) dt - s dt [J_O - J_V]$$

Give a verbal justification of these two equations (and do not forget to give the definitions of J_V and J_O).

- c. (1 points) You should skip this and simply use the result given below, unless you really have extra time at the end of the exam. It is tedious math and will get you only one point. (24) and (25) can be considered two equations in the two unknowns J_O and J_V . Show that solving these equations gives

$$(26) \quad J_V = \frac{(p - w)q(\theta) - \gamma_0(r + s)}{(r + q(\theta))(r + s) - q(\theta)s}$$

$$(27) \quad J_O = \frac{(p - w)(r + q(\theta)) - \gamma_0 s}{(r + q(\theta))(r + s) - q(\theta)s}$$

- d. (5 points) Free entry means that $J_V = 0$. Show that this implies

$$(28) \quad w = p - \frac{(r + s)\gamma_0}{q(\theta)}$$

and

$$(29) \quad J_O = \frac{\gamma_0}{q(\theta)}$$

- e. (3 points) The present value of the expected incomes of employed workers (Y_E) and of unemployed workers (Y_U) can be derived from the following arbitrage conditions:

$$(30) \quad rY_U dt = z dt + \theta q(\theta) dt [Y_E - Y_U]$$

$$(31) \quad rY_E dt = w dt - s dt [Y_E - Y_U]$$

where z is the unemployment benefits. Explain these conditions verbally.

- f. (1 points) Here is another part of the question which only involves tedious math. Don't derive the following two equations unless you have lots of extra time. Just use the results results given here for the other parts of the question. Solving (30) and (31) for rY_U and rY_E gives:

$$(32) \quad rY_U = \frac{(r + s)z + \theta q(\theta)w}{r + s + \theta q(\theta)}$$

$$(33) \quad rY_E = \frac{sz + (r + \theta q(\theta))w}{r + s + \theta q(\theta)} = \frac{r(w - z)}{r + s + \theta q(\theta)} + rY_U$$

- g. (2 points) Firm i takes the product price p and the interest rate r as given, but it does not take the wage w_i as given from the market. Instead, the wage is determined by individual bargaining between firm and worker. After firm i has hired a worker with wage w_i , the present discounted value of its income stream J_O^i satisfies an arbitration equation analogous to (25):

$$(34) \quad rJ_O^i dt = (p - w) dt - s dt [J_O^i - J_V]$$

Write down the Lagrangian and the first-order conditions for this minimization, and show that

$$(43) \quad \pi = -\frac{\alpha}{\beta}[y - y^*]$$

Problem 7. (5 points) Which implications does the Rational Expectations Market Clearing assumption have for policies? Discuss also the issues of time inconsistency.

Problem 8. What is New Keynesian Economics? Explain menu costs, staggering of prices, coordination failure, and efficiency wages. What are the policy implications of New Keynesian Economics?

REFERENCES

- [HVdP02] Ben J. Hejdra and Frederick Van der Ploeg. *Foundations of Modern Macroeconomics*. Oxford University Press, 2002. 1
- [Pis00] Christopher A. Pissarides. *Equilibrium Unemployment Theory*. MIT Press, Cambridge, MA and London, second edition, 2000. 4
- [Sar87] Thomas J. Sargent. *Macroeconomic Theory*. Academic Press, second edition, 1987. 1
- [SW76] Thomas J. Sargent and Neil Wallace. Rational expectations and the theory of economic policy. *Journal of Monetary Economics*, 2:169–183, 1976. 1

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