There are 100 points possible on this exam, 50 points each for Prof. Lozada's questions and Prof. Dugar's questions. Each professor asks you to do two long questions (worth 20 points each) and one short question (worth 10 points each). However, Prof. Lozada gives you a choice of which of his short questions to do, whereas Prof. Dugar gives you a choice of which of his long questions to do.

There are three sections on this exam:

- In the first section contains all of the required questions. There are three of them. The first is worth 20 points; the second is worth 20 points; and the last one is worth 10 points.
- In the second section there are three questions; you should work two of them. Each is worth 20 points.
- In the third section there are two questions; you should work one of them. Each is worth 10 points.

You have 4 hours and 30 minutes (that is, until 1:30 PM) to finish this test. This gives you about 45 minutes per question.

Do not use different colors in your answers because we grade looking at black-and-white photocopies of your exam.

It would be helpful for you to put the number of the problem you are working on at the top of every page of your answers, so we do not accidentally ignore part of your answer.

In this document, v and w respectively denote the Roman lower-case "v" and "w." Also, in this document some questions begin on one page and end on the next page; therefore, do not assume that a question ends at the bottom of a page, but check to determine whether it continues onto the next page.

Good luck.

Section 1.
Answer all of the following three questions.

1. **[20 points]** This question concerns a "Robinson Crusoe" economy.

   (a) First consider Robinson Crusoe the consumer. Assume he obtains utility from consumption of bananas, "b," and hours of leisure, "z," according to the utility function $2\sqrt{b} + 2\sqrt{z}$.

   Suppose "L" represents his number of hours working per day ("labor time"). Assume he earns wage income at a rate of w per hour. Also assume he receives income π from his ownership of the firm.

   The fact that there are only 24 hours in a day constrains z and L. Suppose the price of bananas is p. Suppose Robinson takes p and the wage rate w as given (so he acts in a perfectly competitive way).

   What is his demand for bananas?

   (b) Next consider Robinson Crusoe the firm. Suppose he produces bananas b from labor L according to the production function $b = L^2$.

   i. What kind of returns to scale is this production function?

   ii. Suppose Robinson:

   A. takes the wage rate w as given (so the labor market is competitive); but

   B. *does not* take the price of bananas p as given. Instead, he can control p. Also, he knows everything about the answer to part (a) of this question except he does not know that the π appearing in the consumer's problem is the same as the firm's profit (Robinson-the-firm treats the π in the consumer's problem as just some exogenous constant), and Robinson functions as a monopoly seller of bananas.

   Taking the wage rate w as the numéraire, express the firm's profit as a function of p, instead of taking the usual approach of expressing the firm's profit as a function of L.

   Hint 1: A monopolist's total revenue is "price times output," just as for a competitive firm. However, the firm can pick its p, and it knows the demand curve for b given in the answer to part (a)(i) above.

Hint 2: As an intermediate step, it might be helpful to express the firm's profit as a function of b.

iii. Implicitly find the profit-maximizing level of p. It is completely acceptable to leave this as a system of two equations in two unknowns which you do not try to solve and which you do not try to simplify very much (though you should evaluate all derivatives). Of the two unknowns, one of the unknowns should be p.

2. **[20 points]** Suppose a competitive, profit-maximizing firm transforms two inputs (x_1 and x_2) into an output, which is apples (denoted by a), according to a well-behaved, concave, fully differentiable production function $f(x_1, x_2)$. Let the price of the inputs be p_1 and p_2 and let the price of apples be p_a.

   Suppose that the government introduces an *ad valorem* tax t on the price of apples. In other words, this tax would be expressed as a percentage such as 7%, not as something like $0.70/unit (which would be a "specific tax").

   Feel free to use abbreviations to simplify the answers you derive below.

   (a) How will a change in the price of x_1 affect the demand for x_1?

   (b) How will a change in the price of x_1 affect the demand for x_2?

   (c) How will a change in the price of x_1 affect the supply of a?

   (d) How will a change in the price of x_2 affect the demand for x_1?

   (e) How will a change in the price of x_2 affect the demand for x_2?

   (f) How will a change in the price of x_2 affect the supply of a?

   (g) How will a change in the price of a affect the demand for x_1?

   (h) How will a change in the price of a affect the demand for x_2?

   (i) How will a change in the price of a affect the supply of a?

   (j) Now suppose that all competitive firms producing a use x_1 and x_2 and are subject to this tax on x_1. Using Cramer's Rule, derive an expression for the effect of this tax on the equilibrium price of x_1 when all three prices are allowed to change.

   Your answer will involve 3×3 determinants; you should leave them unevaluated to save time. As an additional time-saving measure, just assume the number of firms producing a is equal to one even though that is a strange assumption because the firm(s) is (are) competitive.

(A similar question appeared on a previous exam in a past year, and the answer I gave for it only involved a 2×2 determinant, but that answer should have taken one more market into account, and if it had done so, it would have involved a 3×3 determinant as well.)

3. **[10 points]** Consider the following game. The two players 1 and 2 have preferences over two goods, x and y. 1 is endowed with one unit of good x and none of good y, while 2 is endowed with one unit of good y and none of good x. Each player i ($i = 1, 2$) has utility function of the form: $\min\{x_i, y_i\}$, where x_i is i's consumption of x and y_i is i's consumption of y. The game proceeds as follows. Each player simultaneously hands any (nonnegative) quantity of the good he possesses (up to his entire endowment) to the other player.

   (a) Represent the above strategic situation as a normal-form game.

   (b) Find all pure-strategy Nash equilibria of this game.

   (c) Does this game have a dominant strategy equilibrium? If so, what is it? If not, why not?

Section 2.
Answer two of the following three questions.

1. **[20 points]**
Consider the following two-player (a proposer called P and a responder called R) bargaining game. The proposer has \$1. The proposer offers a share s between \$0 and \$1 to the responder. If the responder accepts the offer, then the proposer's offer is implemented and the payoffs of the proposer (π_P) and the responder (π_R) are, respectively, $(1 - s)$ and s. If the responder rejects the offer, the respective dollar payoffs are 0 and 0. The game is played only once. Suppose instead of selfish preferences, both players have "inequity averse" preferences, due to Fehr-Schmidt (1999). The Fehr-Schmidt utility function (or FS utility function) of an individual i ($i = P, R$) is given by:

$$U_i(\pi_i, \pi_j) = \pi_i - \alpha_i \max\{\pi_j - \pi_i, 0\} - \beta_i \max\{\pi_i - \pi_j, 0\} \quad \text{for } j \neq i$$

where $\alpha_i \geq 0$, $0 \leq \beta_i < 1$ and $\beta_i < \alpha_i$. All of the above is common knowledge between the players. One can think of α as a parameter capturing i's "envy" and β as a parameter capturing i's "guilt."

(a) Assuming players are selfish, what is the subgame perfect Nash equilibrium of the game? Write down all the assumptions that you need to make to answer this question.

(b) Assume that players have inequity averse preferences. Show all of the following: the responder accepts all offers $s \geq 0.5$. There is a critical share, $s_c < 0.5$, such that: (1) the responder rejects all offers $s < s_c$; (2) the responder accepts all offers $s \geq s_c$; and (3) the critical share s_c is increasing and strictly concave in α_R.

(c) Assume that players have inequity averse preferences. Prove that the equilibrium share (s^*) offered by the proposer to the responder is given by the following: $s^* = s_c$ if $\beta_P < 0.5$; $s^* = 0.5$ if $\beta_P > 0.5$; $s^* = s \in [s_c, 0.5]$ if $\beta_P = 0.5$.

(d) Now suppose that players are selfish. Moreover, there are $n > 2$ players, where the first $n - 1$ players are proposers and the n^{th} player is the single responder in the game. The proposers simultaneously make an offer to the responder. All other details of the game remain the same. Prove that in any subgame perfect outcome, at least two proposers must offer \$1, the maximum offer among all the proposers, which is accepted by the responder.

2. **[20 points]**
Consider a strictly risk-averse decision maker who has an initial wealth of w but who runs a risk of a loss of D dollars. The probability of the loss is π. It is possible, however, for the decision maker to buy insurance. One unit of insurance costs q dollars and pays 1 dollar if the loss occurs. The decision maker's problem is to choose the optimal amount of insurance, denoted by α.

   (a) Show that if insurance is not actuarially fair (so that $q > \pi$) then the decision maker will not insure completely.

   (b) Suppose that the decision maker has a logarithmic utility function. Derive the optimal level of coverage when $q = \pi$.

   (c) To continue with the assumption of a logarithmic utility function, derive an expression showing the relationship between α and q. Under what condition is insurance a Giffen good?

   (d) Suppose the insurance market is perfectly competitive and the decision maker has a logarithmic utility function. Furthermore, the decision maker's wealth, w, increases. Does the decision maker increase the demand for insurance? Show full mathematical work. Argue in favor of your answer. How would your answer change if the insurance was actuarially unfair?

3. **[20 points]**
Consider the following two-player signaling game. There is a pot, which initially has $1, and which will be awarded to one of the players. Each player i privately draws a card, where the value v_i of player i's card is distributed $U[0, 1]$. Player 1 moves first, choosing to "bid" or "fold". If he bids, then he must add $1 to the pot. Player 2 then gets the move and must either "Bid" or "Fold". If he chooses Bid, then he must add $1 to the pot. The players then show their cards and the player with the highest card wins the contents of the pot. If either player folds, then the game ends immediately and the other player wins the pot.

   If, for example, both players bet and player 1 has the highest card, then Player 1 wins $2 (and receives back the $1 he put in the pot), and Player 2 loses $1.

   (a) In this game, what is a strategy for Player 1? For Player 2? Allow for mixed strategies also.

(b) Find a Perfect Bayesian equilibrium of this game. Hint: Look for an equilibrium in which Player 1 bets if $v_1 \geq c_1$ and folds if $v_1 \leq c_1$, where c_1 is a constant.

Section 3.
Answer one of the following two questions.

1. **[10 points]**

   (a) A firm has two plants with cost functions $c_1(y_1) = y_1^2/2$ and $c_2(y_2) = y_2$.

      i. Sketch the marginal cost curves of the two plants on one graph.

      ii. What is the cost function of the firm?

      iii. According to your cost function, how much would it cost this firm to produce an output of $1/4$? (If you got part (ii) right, this question is so easy that you may wonder why I ask it. The reason I ask it is that if you got part (ii) wrong, this question may reveal your error to you.)

   (b) A firm has two plants. One plant produces output according to the production function $x_1^a x_2^{1-a}$. The other plant has a production function $x_1^b x_2^{1-b}$.

      i. What is the cost function for this technology?

      ii. If this firm wanted to produce a total output of 2, how much would be produced in each plant?

2. **[10 points]** Consider a partial equilibrium in the market for apples, with the following (aggregate) supply and demand curves:

$$D(p) = 10 - p$$
$$S(p) = 3p + 1 \,.$$

   (a) Find the equilibrium price and quantity of apples the way most undergraduate students would do it, by solving the two equations in two unknowns.

   (b) Find the equilibrium price of apples by formulating this as a fixed-point problem. Then determine the equilibrium quantity of apples. (Do not determine the equilibrium quantity of apples first.)

   (c) Find the equilibrium quantity of apples by formulating this as a fixed-point problem. Then determine the equilibrium price of apples. (Do not determine the equilibrium price of apples first.)