Summer 2012 Microeconomics Qualifying Exam

There are 72 points possible on this exam, 36 points each for Prof. Lozada's questions and Prof. Kiefer's questions. However, Prof. Lozada's questions are weighted differently from Prof. Kiefer's questions: Prof. Lozada's questions are worth 14 points, 14 points, and 8 points, while Prof. Kiefer's questions are worth 18 points, 9 points, and 9 points.

There are three sections on this exam:

- In the first section there are three questions; you should work all of them. The first is worth 14 points; the second is worth 14 points; and the last one is worth 18 points.
- In the second section there are two questions; you should work one of them. Each is worth 8 points.
- In the third section there are three questions; you should work two of them. Each is worth 9 points.

You have 4 hours and 30 minutes (that is, until 1:30 PM) to finish this test. This gives you about 45 minutes per question.

Do not use different colors in your answers because we grade looking at black-and-white photocopies of your exam.

It is helpful (but not required) if you put the number of the problem you are working on at the top of every page.

Good luck.

Section 1.
Answer all of the following three questions.

1. [**14 points**] Suppose an economy has two consumers, Smith and Jones, and two commodities, x and y. Smith's utility function and initial endowment are

$$u_s = \ln x_s + \ln y_s$$
$$\boldsymbol{\omega}_s = (\omega_{sx}, \omega_{sy}) = (0, 1)\,.$$

Jones's initial endowment is

$$\boldsymbol{\omega}_j = (\omega_{jx}, \omega_{jy}) = (1, 0)\,.$$

So the only way Jones can get any of good y is to get it from Smith. Jones's true utility function is

$$u_j = \ln x_j + \ln y_j$$

but he may be unsure about the quality of the y which he gets from Smith; we will model this by assuming Jones's utility function is instead

$$u_j = \ln x_j + \psi \ln y_j$$

where $0 \leq \psi \leq 1$.

   (a) What situation does $\psi = 1$ represent?

   (b) What situation does $\psi = 0$ represent?

   (c) Find the general equilibrium x_j, y_j, x_s, and y_s if $\psi > 0$.

   (d) Find the general equilibrium x_j, y_j, x_s, and y_s if $\psi = 0$. Be sure your answer makes intuitive sense.

   (e) Set up an Edgeworth Box Diagram. Show on this diagram how the allocations of x and y to Smith and Jones change as ψ changes from 1 to 0. (It makes no sense to draw indifference curves on this diagram because changing ψ implies changing utility functions.)

2. [**14 points**] Suppose a person consumes two goods, x and y. The price of x is \$0.50 per unit (that is, \$1/2 per unit). The price of y reflects a volume discount, and is

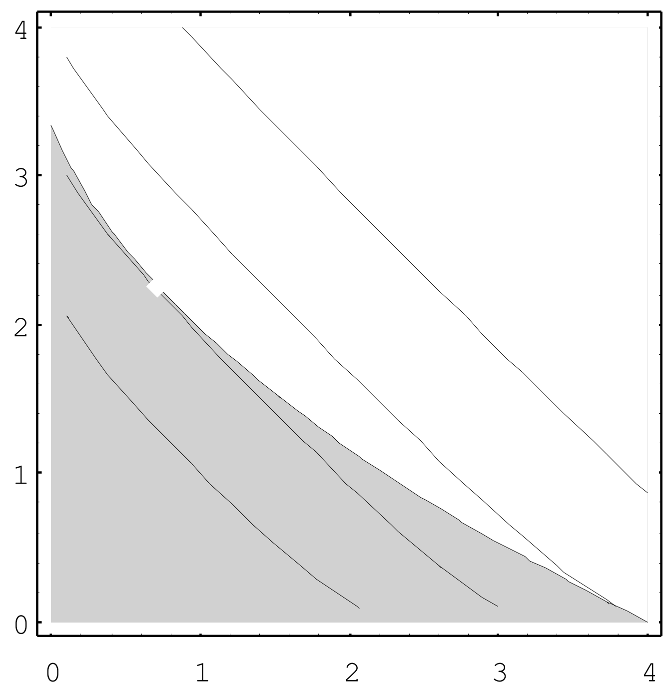
$$1 - 0.12y$$

as long as that is positive. The consumer's income is \$2. This consumer's affordable set is the shaded area in the graph below.

Suppose this consumer's utility function is

$$x + y + \tfrac{1}{10} \ln x + \tfrac{1}{10} \ln y\,.$$

Some of this consumer's indifference curves are shown in the graph below.



(a) Show that the utility function is strictly concave.

(b) Show that any (x^*, y^*, λ^*) satisfying

$$
\begin{aligned}
2 &= -\frac{1}{10} \cdot \frac{2.4y^2 - 10y}{4.8y^2 - 10y + 1} + y - 0.12y^2 \\
x &= -\frac{1}{5} \cdot \frac{2.4y^2 - 10y}{4.8y^2 - 10y + 1} \\
\lambda &= 2 + \frac{1}{5x}
\end{aligned}
$$

(or an equivalent set of equations) satisfies the first-order conditions for utility maximization. (These equations should have been written as functions of x^*, y^*, and λ^*, but I omitted the asterisks for enhanced legibility.) Do not try to solve the system for y^*, x^*, or λ^*.

(c) What sufficient condition would ensure that a vector (x^*, y^*, λ^*) satisfying the conditions of part (b) actually is a maximum? Your answer should be a function of x^*, y^*, and λ^*, but you can omit the asterisks for enhanced legibility.

(d) It can be shown that $(x^*, y^*, \lambda^*) = (0.704259, 2.26171, 2.28399)$ satisfies the conditions of part (b). This point is marked as a dot on the graph. However, it violates the condition of part (c) (do not prove this; take my word for it). What is the implication of this violation? Could you have predicted this violation?

(e) What do you guess the consumer's utility-maximizing bundle is? Why? (I am asking for a *guess* here, not a mathematical investigation.)

3. **[18 points]**

Kim, Khloe and Kourtney share an apartment at Quasilinear Gardens. They each derive benefit from apartment cleanliness G, a public good, and leisure x, a private good. Their utility functions and endowments are

$$\begin{array}{lll} \text{Kim,} & U_i = x_i + \frac{1}{6}\ln G, & \omega_i = 1 \\ \text{Khloe,} & U_h = x_h + \frac{1}{3}\ln G, & \omega_h = 1 \\ \text{Kourtney,} & U_o = x_o + \frac{1}{2}\ln G, & \omega_o = 1. \end{array}$$

Each citizen may make a contribution g toward the provision of cleanliness, but such contributions reduce private consumption according to the budget constraint

$$\omega = g + x.$$

Production of cleanliness and leisure takes place according to the transformation function,

$$T(G, x_i, x_h, x_o) = 0 = G + x_i + x_h + x_o - \omega_i - \omega_h - \omega_o.$$

(a) Find the Nash equilibrium; express your answer as $\big(G, x_i, x_h, x_o\big)$.

(b) Find the Lindahl equilibrium.

(c) Find the Bowen equilibrium.

(d) Which of these three equilibria does a Rawlsian social planner prefer?

(e) The Lindahl and Bowen equilibriums both have the same value of G. What conditions in general are necessary to guarantee this outcome? Show that they apply in this case. In what ways do Lindahl and Bowen still differ?

(f) Show that the Bowen equilibrium is not Pareto superior to the Nash equilibrium. Find a potential redistribution of the Bowen equilibrium that makes all three better off.

(g) Using the alternative social choices $G = 1/2$, $G = 1$, and $G = 2$, explain why a voting paradox does not arise in this Bowen democracy. Could this conclusion change with a different set of public goods? (Note that $\ln(1/2) \approx -0.7$, $\ln 1 = 0$, and $\ln 2 \approx +0.7$.)

(h) Discuss the wider implications of this example.

Section 2.
Answer one of the following two questions.

1. [**8 points**] In Utah:

   - Both farmers and people who live in cities use water supplied by government water companies. There are different government water companies for cities and for farms.
   - Farmers pay a much lower price for water than city dwellers do.
   - In the future, farmers might start reselling the water they buy to city dwellers. (Ignore why they do not do this now.)
   - The price that city dwellers pay for water is lower than in neighboring states because Utah city water companies also get a great deal of money from taxes which have nothing to do with water. These taxes are called "property taxes."
   - The Utah legislature is considering eliminating the flow of money from "property taxes" to the city water companies.

   Question: If Utah legislature did eliminate the flow of money from "property taxes" to the city water companies, would this make the price of water sold by farmers to city dwellers go up or down? (No water is sold by farmers to city dwellers right now, but ignore that.) In particular, analyze in detail each claim made in the following paragraph written by an economics professor at another university in Utah, and explain whether you think his analysis is correct or not:

   > "I don't see why the property tax will affect much the value of water used in agriculture. A higher urban price because of the elimination of the tax subsidy will induce conservation that will reduce the urban demand for new water. This will reduce the demand for ag[ricultural] water and may reduce the equilibrium transfer price. Therefore, we might expect opposition from farmers to eliminating the [property] tax subsidy."

   Hint: It is possible to correctly answer this question by drawing a graph but using no other mathematics.

2. [**8 points**] Suppose a competitive firm produces output q from inputs x_1 and x_2 according to the production function

$$q = x_1^{\alpha} x_2^{\beta}$$

where $\alpha > 0$ and $\beta > 0$. Assume the second-order sufficient conditions for the firm's problem are satisfied.

By how much would the firm's choice of x_1 change if α rose slightly? Attempt to sign this derivative.

Hint 1: If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then

$$A^{-1} = \frac{1}{|A|} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

Hint 2:

$$\frac{d}{dx} a^x = a^x \ln a, \text{ not } x a^{x-1}.$$

Hint 3: It might be easiest to use Cramer's Rule at some point in answering this question.

Section 3.
Answer two of the following three questions.

1. **[9 points]**
   The basketball coaches of BYU and Utah are both trying to recruit the same local high school star, LeBron Bryant. Both estimate that LeBron will generate $3 million in revenue (after deducting the cost of his scholarship) over the next two years. LeBron is considering only these two schools, and will not join the NBA draft before his junior year (due to NBA regulations). However, before committing LeBron demands that the coach arrange for a secret deposit of $1 million into his Swiss bank account. Of course, this bribe would be against NCAA recruiting rules, and punishable by severe sanctions. However, each coach knows that there is zero chance that the NCAA will detect the violation (wealthy, but discreet boosters will pay the bribes).

   Each coach also knows that he is certain to sign LeBron if he pays the bribe, while his rival refuses to bribe. Each coach knows that his chance of signing is even (50:50) when both rivals follow the same strategy. Each also knows that if both offer to bribe, then the losing school will not actually pay the bribe.

   The payoff table below describes the situation.

<table>
<tr><td colspan="2" rowspan="2">expected payoffs
(Utah, BYU)</td><td colspan="2">BYU</td></tr>
<tr><td>violate rules</td><td>obey rules</td></tr>
<tr><td rowspan="2">University of Utah</td><td>violate rules</td><td></td><td></td></tr>
<tr><td>obey rules</td><td></td><td></td></tr>
</table>

   (a) Fill in the payoff matrix. Explain.

   (b) Consider a single simultaneous game. Do any players have dominant strategies? Is there a Nash equilibrium for the one-shot game?

   (c) Now consider an infinite number of repetitions. Under what conditions is the (obey, obey) outcome a subgame-perfect Nash equilibrium? Discuss the role of information on the repeated-game equilibrium.

2. **[9 points]**
Khloe and Lamar consume a private good, coffee x_i, and a public good, poetry G. The utility functions and endowments are given as follows:

$$\begin{array}{lll} \text{Khloe} & U_k = \min(x_k, G) & \omega_k = 4, \\ \text{Lamar} & U_l = \min(x_l, G) & \omega_l = 2. \end{array}$$

Each citizen may make a contribution g toward the provision of poetry, but such contributions reduce private consumption according to the budget constraint

$$\omega = g + x.$$

The private good can be transformed into the public one according to the transformation function

$$T(G, x_l, x_k) = 0 = G + x_l + x_k - \omega_l - \omega_k.$$

Finally, Khloe and Lamar agree on the Benthamite social welfare function,

$$W = U_k + U_l.$$

(a) Plot reaction curves in g_k — g_l space. Find the Nash equilibrium.

(b) Add indifference curves to your g_k — g_l diagram. Find all Pareto efficient allocations.

(c) Show that the Nash Equilibrium is also the Benthamite social optimum. Illustrate your answer in U_k — U_l space.

3. [**9 points**]
Comcast and Direct TV produce TV broadcasts x_i by combining capital k and labor l. Their cost functions are

$$c(v_i, w_i, x_i) = (v_i + w_i)x_i \,,$$

where v is the rental rate for capital, w the wage rate and $i = c$ or d. Suppose that inverse demand is given by $p(x) = 4 - x$, where x is the total quantity of TV broadcasts.

(a) Suppose these two firms share the market, and that they behave as a Cournot duopoly. Both face the same costs in competitive factor markets. The rental rate for capital is $v_i = 1$; the wage rate is $w_i = 1$. What is the equilibrium? Illustrate your answer with best response curves.

(b) Now suppose that Comcast's workers unionize and succeed in raising their wage to $w_c = 2$. Find the new equilibrium.

(c) Construct a welfare analysis of these two equilibriums, (a) and (b), to determine which is the more efficient market structure. Calculate the monetary value of the difference.