There are 72 points possible on this exam, 36 points each for Prof. Lozada's questions and Prof. Kiefer's questions. However, as you can see below, Prof. Lozada's questions are equally weighted, while Prof. Kiefer's required question is worth twice as much as his optional questions.

There are three sections on this exam:

- In the first section there are three questions; you should work all of them. The first two are worth 12 points each; -the last-one is worth 18 points.
- In the second section there are two questions; you should work one of them. Each is worth 12 points.
- In the third section there are three questions; you should work two of them. Each is worth 9 points.

You have 4 hours and 30 minutes (that is, until 1:30 PM) to finish this test. This gives you about 45 minutes per question. Good luck.

# Section 1. Answer all of the following three questions.

### 1. [12 points]

There is an old saying,

"Idle hands are the devil's workshop."

In other words, excessive idleness ("leisure") is a bad thing for people. Suppose a price-taking consumer's utility depends on his purchases of a good  $x \ge 0$  and his consumption of leisure z (the notation "z" recalls the sound at the beginning of the second syllable of "leisure").

(a) Argue that postulating a utility function of

$$u = \ln x + [-(z-1)^2 + 1]$$
 with  $0 \le z \le 2$ 

is a reasonable way of modeling the idea that excessive idleness is bad.

- (b) Suppose the wage rate (that is, the payment for the opposite of leisure) is w. Suppose the price of x is p. What is the consumer's budget constraint?
- (c) From the first-order conditions, argue that in order for the optimal x to be positive, the optimal z must be in (0,1).
- (d) Find the optimal x and z in terms of exogenous variables.
- (e) What does this consumer's labor supply curve look like?
- (f) Check the second-order conditions for the optimization problem.
- (g) What is this consumer's indirect utility function?

## 2. [12 points]

- (a) Are individual firm's supply curves always upward-sloping? Usually? Never? Defend your answer with as general a mathematical argument as you can provide.
- (b) Are individual demand curves always downward-sloping? Usually? Never? Defend your answer with as general a mathematical argument as you can provide.
- (c) The US federal government imposes a tax on gasoline. Gasoline has recently greatly risen in price, and two presidential candidates, Sen. Hillary Clinton and Sen. John McCain, have proposed eliminating this tax this summer. The other major presidential candidate, Sen. Barack Obama, opposes eliminating the tax this summer. Irrelevant to this qualifying exam question are most of Sen. Obama's reasons (such as encouraging alternative energy sources), but one reason is relevant: some of Sen. Obama's supporters have said that if the tax is eliminated, the gasoline companies will just raise the price by the amount of the eliminated tax, so the "net price to consumers" will not change.

Is this true?

To answer this, derive a general mathematical equation showing how a marginal decrease (or marginal increase) of a tax changes the market price of a commodity. Will a marginal decrease of a tax increase, decrease, or have no effect on the "net price to consumers?"

Hint: Recall that 19th century Scottish historian Thomas Carlyle (who also coined the term "dismal science" to denote economics) said, "Teach a parrot the terms 'supply and demand' and you've got an economist." So, begin there (mathematically), in a market with no tax.

## 3. [18 points]

Mac produces burgers y by combining grills  $x_1$  and labor  $x_2$ .

(a) Mac's cost function is

$$c(w_1, w_2, y) = \min(2w_1, w_2)y$$

where  $w_1$  is the rental rate for grills and  $w_2$  the wage rate. What is his production function? Is it homogeneous of degree 1? Monotonic? Convex? Plot his isocost and isoquant curves; give an interpretation the slopes for this case.

(b) Suppose that the rental rate for grills is  $w_1 = 1$ ; the wage rate is  $w_2 = 2$ , and that demand is given by the function

$$p(y)=4-y.$$

And, suppose that this market is competitive. What is its equilibrium? What combination of grills and labor does Mac use?

- (c) Now suppose two firms (Mac and Wendy) share this market; they behave as a Cournot duopoly. They have identical costs. What is the equilibrium?
- (d) Mac and Wendy still share the market, but now form a cartel. Suppose that they split the cartel quantity and profits equally. And assume that the cartel and Cournot quantities are the only possibilities. Fill in the payoff matrix.

payoffs: (Mac, Wendy)		Wendy	
		cartel	Cournot
Mac	cartel		
	Cournot		

- (e) Suppose that this game is repeated infinitely many times. Consider the *mutual punishment* threat:
  - play cartel in the first game; thereafter play cartel,
  - unless the either rival plays Cournot in any previous game, then play Cournot.

What outcome will occur if both players follow this strategy? Under what circumstances is the (cartel, cartel) outcome a subgame-perfect Nash equilibrium of an infinitely repeated game?

- (f) Finally, suppose that these two firms behave as a Stackelberg duopoly: only one game is played, Mac chooses his quantity first, and Wendy follows. What is Stackelberg equilibrium? Is this a Nash equilibrium?
- (g) Construct a welfare analysis of the cartel, Cournot and Stackelberg equilibriums to determine which is the more efficient market structure. Illustrate your answer. What are the implications of this example for public policy?

# Section 2. Answer one of the following two questions.

### 1. [12 points]

Baby Alice has two grandfathers, Grandpa Smith and Grandpa Jones. Suppose there is one good, called "x", and suppose the consumption of that good by Baby Alice, Grandpa Smith, and Grandpa Jones is  $x_A$ ,  $x_S$ , and  $x_J$ , respectively.

Grandpa Smith has an endowment of 3 units of good x, of which he consumes  $x_S$  and gives  $x_{SA}$  to Baby Alice:

$$x_S + x_{SA} = 3.$$

Grandpa Jones has an endowment of 3 units of good x, of which he consumes  $x_J$  and gives  $x_{JA}$  to Baby Alice:

$$x_J + x_{JA} = 3.$$

Baby Alice has no endowment of good x; all of her consumption consists of gifts from her grandfathers:

$$x_A = x_{SA} + x_{JA}.$$

Baby Alice's utility function is

$$u_A(x_A) = \ln x_A .$$

Grandpa Smith's utility function is

$$u_S(x_S, u_A) = \ln x_S + u_A.$$

Grandpa Jones's utility function is

$$u_J(x_J, u_A) = \ln x_J + u_A.$$

(So the grandfathers care about their own consumption and the utility of Baby Alice.)

(a) Find the values of  $x_{SA}$ ,  $x_{JA}$ ,  $x_A$ ,  $x_S$ ,  $x_J$ ,  $u_S$ ,  $u_J$ , and  $u_A$ . Assume that each grandfather takes the other grandfather's actions as fixed. (So this is technically an equilibrium in the sense of Nash.)

- (b) Denote the values found in part (a) as  $x_{SA}^*$ ,  $x_{JA}^*$ ,  $x_A^*$ ,  $x_S^*$ ,  $x_J^*$ ,  $u_S^*$ ,  $u_J^*$ , and  $u_A^*$ . Now suppose that instead of giving  $x_{SA}^*$  and  $x_{JA}^*$ , each grandfather increases his gift by  $\epsilon > 0$ . Show that for sufficiently small  $\epsilon$ , this change increases everyone's utility compared with the situation of part (a).
- (c) Is the Nash Equilibrium efficient?
- (d) Briefly speculate about the welfare implications of this example if, instead of Baby Alice having two grandfathers, she had four great-grandfathers.

#### 2. [12 points]

Suppose a price-taking firm uses inputs  $x_1$  and  $x_2$  to produce output y according to the production function  $y = \sqrt{x_1} + \sqrt{x_2}$ .

- (a) Find this firm's cost function.
- (b) Do the second-order conditions for this problem hold?
- (c) How would the analysis in (a) and (b) change if the firm were not a price taker?

# Section 3. Answer two of the following three questions.

## 1. [9 points]

The postulate of methodological individualism underlies all public choice analysis. In trying to explain governmental actions, we begin by analyzing the behavior of the individuals who make up the government. In a democracy these are the voters, their elected representatives, and appointed bureaucrats. The postulate of methodological individualism has a normative analogue. The actions of government ought to correspond, in some fundamental way, to the preferences of the individuals who these actions effect, the citizens of the state. The postulate of normative individualism underlies much of normative analysis in public choice.

- —Dennis Mueller
- (a) Explain the term methodological individualism. Discuss its role in neoclassical microeconomics. Give specific examples.
- (b) In light of the assertion,

"Democracy is imperfect, but better than the alternative,"

discuss the limitations of normative individualism.

(c) Are there alternative schools of thought with respect to methodology? Discuss and evaluate.

## 2. [9 points]

Question 2. Consider a duopoly strategy game with three options labeled *left*, *middle* and *right*. The profit payoff matrix is

profit pay	offs:	Lowe's		
(Home Depot, Lowe's)		left	middle	right
	left	10,10	5,12	0, -1
Home Depot	middle	12,5	7,7	1,0
	right	-1,0	0,1	2,2

- (a) In a one-shot, simultaneous game, explain why the *left* strategy is dominated for both players. Are there any Nash equilibriums? Explain.
- (b) Two games are played; each is simultaneous. Consider a strategic threat:
  - play left in the first game,
  - if rival plays *left* in the first, then play *middle* in second; else play *right* in second

Under what circumstances is (1st: left, 2nd: middle) a Nash equilibrium for both firms?

(c) Is (1st: left, 2nd: middle) subgame perfect?

## 3. [9 points]

Barbie and Ken consume a private good, coffee  $x_i$ , and a public good, poetry G. The utility functions and endowments are given as follows:

Barbie 
$$U_b = \min(x_b, G)$$
  $\omega_b = 6$ ,

Ken 
$$U_k = \min(x_k, G)$$
  $\omega_k = 3$ .

Each citizen may make a contribution  $g_i$  toward the provision of poetry, but such contributions reduce private consumption according to the budget constraint

$$\omega_i = x_i + g_i.$$

The private good can be transformed into the public one according to the transformation function

$$x_b + x_k + G - \omega_b - \omega_k = 0.$$

Finally, Barbie and Ken agree on the Benthamite social welfare function,

$$W=U_k+U_b.$$

- (a) Plot reaction curves in  $g_k g_b$  space. Find the Nash equilibrium.
- (b) Add indifference curves to your  $g_k g_b$  diagram. Given these endowments, find all Pareto efficient allocations.
- (c) Show that the Nash Equilibrium is also the Benthamite social optimum, given these endowments. Illustrate your answer in  $U_k U_b$  space.