Summary 2006 | Microeconomics Qualifying Exam

There are 72 points possible on this exam, 36 points each for Prof. Lozada's questions and Prof. Kiefer's questions. However, as you can see below, Prof. Lozada's questions are equally weighted, while Prof. Kiefer's required question is worth twice as much as his optional questions.

There are three sections on this exam:

- In the first section there are three questions; you should work all of them. The first two are worth 12 points each; the last one is worth 18 points.
- In the second section there are two questions; you should work one of them. Each is worth 12 points.
- In the third section there are three questions; you should work two of them. Each is worth 9 points.

You have 4 hours and 30 minutes (that is, until 1:30 PM) to finish this test. This gives you about 45 minutes per question.

Good luck.

Section 1.
Answer all of the following three questions.

1. **[12 points]**
Suppose a competitive firm earns profit π by producing output Q using a cost function $C(Q)$ with $Q \geq 0$.

   (a) Under what conditions is this firm's supply curve upward-sloping?

   (b) Suppose the cost function has the form

$$C(Q) = 4\sqrt{Q} + \frac{\sqrt{8}}{2}Q^2$$

   (a graph of this function is attached to this exam). Find the firm's supply curve. You do not have to find it explicitly; implicitly is enough. Be sure you work through the following steps to help ensure your answer is right.

      i. For what values of Q are the appropriate second-order conditions satisfied?

      ii. For the values of Q you found in (i), what are the corresponding values of p?

      iii. For other values of p, what is the optimal quantity supplied? (Hint: at other values of p, does the Q which satisfies the first-order condition yield a maximum or a minimum?)

2. **[12 points]**
Consider a two-person, two-good economy with persons named Smith and Jones and goods named 1 and 2. Suppose the amount of goods 1 and 2 that Smith consumes is denoted by x_{1s} and x_{2s}, respectively. Suppose the amount of goods 1 and 2 that Jones consumes is denoted by x_{1j} and x_{2j}, respectively. Suppose Smith's utility function is $u_s = x_{1s}x_{2s}^2$ and Jones's utility function is $u_j = x_{1j}x_{2j}^2$. Suppose the total amount of good 1 in the economy is 3 and the total amount of good 2 in the economy is 4.

   (a) Find the location of the contract curve in the Edgeworth Box of this economy. Suppose the Edgeworth Box has Smith's origin in the lower-lefthand corner, and describe the contract curve as a function of Smith's consumption, x_{1s} and x_{2s}, not of Jones's consumption. Graph Smith's consumption of good 1 on the horizontal axis and Smith's consumption of good 2 on the vertical axis.

   (b) What is the slope of the contract curve? Is it positive or negative?

   (c) What is the second derivative of the contract curve? Is it positive or negative?

   (d) As pointed out on p. 554 of the text by Sydsæter and Hammond, which is attached to this exam, the gradient ∇F of a function F is orthogonal to the tangent of its level curve.

      i. Calculate the gradient of the utility function of Smith at an arbitrary point on the contract curve.

      ii. From this, calculate the slope of the gradient of the utility function of Smith at an arbitrary point on the contract curve.

      iii. Is the answer to part (d)(ii) the same as the slope you found in part (b)?

      iv. Suppose a pair of indifference curves, one for Smith and one for Jones, pass through the same point on the contract curve. At this point, is the tangent line to Smith's indifference curve perpendicular to the contract curve? At this point, is the tangent line to Jones's indifference curve perpendicular to the contract curve?

3. **[18 points]**
Angelina, Brad and Jennifer share an apartment at Quasilinear Gardens. They each derive benefit from apartment cleanliness G, a public good, and leisure x, a private good. Their utility functions and endowments are:

Angelina, "slob,"	$U_a = x_a + \sqrt{G}$,	$\omega_a = 9$,
Brad,	$U_b = x_b + 2\sqrt{G}$,	$\omega_b = 9$,
Jennifer, "neatnik,"	$U_j = x_j + 3\sqrt{G}$,	$\omega_j = 9$.

Production of cleanliness and leisure takes place according to the transformation function,

$$T(G, x_a, x_b, x_j) = G + x_a + x_b + x_j - \omega_a - \omega_b - \omega_j = 0\,.$$

(a) Find the Nash equilibrium; express your answer as (G, x_a, x_b, x_j).

(b) Find the Lindahl equilibrium.

(c) Find the Bowen equilibrium.

(d) The Lindahl and Bowen equilibriums both have the same value of G. In general what conditions are necessary to guarantee this outcome? Show that they apply in this case. In what ways do Lindahl and Bowen still differ?

(e) Show that the Bowen equilibrium is not Pareto superior to the Nash equilibrium, but can be justified according to the compensation criterion.

(f) Which of these three equilibria does a Rawlsian social planner prefer?

(g) Discuss the wider implications of this example.

Section 2.
Answer one of the following two questions.

1. [**12 points**]
Suppose a profit-maximizing firm produces apples from two inputs, x_1 and x_2, according to the production function $f(x_1, x_2)$. Suppose the government imposes an *ad valorem* tax of t dollars per pound on apples.
   (a) Derive an expression showing how the firm's purchases of x_1 change when the tax goes up infinitesimally.
   (b) Find the sign of the expression you derived in part (a). Hint: It may be easiest to do this in an indirect way, arguing the standard Hotelling's Lemma with the standard profit function (whose concavity or convexity you may merely assert; you do not have to prove it).
   (c) Derive an expression showing how the firm's profit changes when the tax goes up infinitesimally. Find the sign of this expression.

2. [**12 points**]
Suppose the consumers in this problem are competitive. This is true for *both* parts (a) and (b) of this question! For simplicity, assume there is only one consumer (this just saves on notation). This consumer only consumes two goods, X and Y. Suppose this consumer's income is \$10, the price of good X is \$2/unit, and the price of good Y is \$1/unit. Suppose the consumer's utility function is $X^{1/2}Y^{1/2}$.
   (a) How much X and Y will this consumer buy? Be sure to verify the second-order conditions.
   (b) Suppose when this consumer goes to the store to buy X and Y, he can only find 4 units of Y in the store. (This should be less than the amount you calculated that he desired to buy in part (a).) What do you think will happen? Bidding the price of good Y up? In the end, how much X and Y will he end up with?

Section 3.
Answer two of the following three questions.

1. **[9 points]**
   Imagine a duopoly game with the following profit payoffs.

payoffs: (Microsoft, Apple)		Apple	
		collude	attack
Microsoft	collude	(10, 2)	(8, 3)
	attack	(9, 1)	(7, 0)

   Think of this as a nonspecific game, not necessarily Cournot or Bertrand. Only two strategies are available.

   (a) Consider a single simultaneous game. Do any players have dominant strategies? Is there more than one Nash equilibrium?

   (b) Now consider an infinite number of repetitions of the simultaneous game. Are there any conditions under which the (collude, collude) outcome is a Nash equilibrium? If (collude, collude) is an equilibrium, is it subgame-perfect?

   (c) Suppose that Microsoft moves first, and that only one game is played. Draw the extensive form of this sequential game. What is the subgame-perfect Nash equilibrium? Discuss.

2. **[9 points]**
Consider a society of 9 citizens who have rankings over three candidates (Al, George, and Ralph) as shown.

ranking	5 voters	4 voters
first	Al	George
second	George	Ralph
third	Ralph	Al

(a) Consider the following social decision rules:

i. What is the Condorcet rule? Who is the "Condorcet winner?"

ii. What is the plurality rule? Who is the "plurality winner?"

iii. What is the Borda count rule? Who is the "Borda winner?"

(b) Which of these rules do Dasgupta and Maskin prefer? Explain.

(c) Relate this case of 9 citizens to May's and Arrow's theorems. Do these theorems favor any of these three rules? Which rule do you prefer, and why?

3. [**9 points**]
Boeing and Airbus produce passenger jets in a duopolistic airplane market. Both have identical marginal and average costs of 1. World demand for jets is given by the function

$$p(x) = 5 - x.$$

(a) Suppose the two firms behave as a Cournot duopoly. Draw their reaction curves. What is the equilibrium price and quantity?

(b) Suppose that the two firms form a cartel. What is the equilibrium price and quantity? How do they divide the profit?

(c) Suppose that they split the cartel quantity and profits equally, as shown below. And assume the cartel and Cournot quantities are the only possibilities. Fill in the rest of the payoff matrix.

payoffs: (Boeing, Airbus)		Airbus	
		cartel	Cournot
Boeing	cartel	(2, 2)	
	Cournot		