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Consumer Choice under Carbon Rationing

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Contents

1. Formulation of the Problem

Throughout this paper we will use the utility function

$$\nu = e\, g\, h \tag{1}$$

where $e \geq 0$ is electricity consumption, $g \geq 0$ is gasoline consumption, and $h \geq 0$ is carbon-free consumption. Consumers maximize ν subject to the budget constraint (2) and the carbon constraint (3).

$$e\, p_e + g\, p_g + h\, p_h \leq m \tag{2}$$

$$e\, q_e + g\, q_g \leq a \tag{3}$$

m is money income and a is the carbon ration, p_e, p_g, and p_h are the prices, and q_e and q_g could be called the carbon footprint of electricity

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and gasoline, i.e., the number of carbon allowances which have to be paid per unit of electricity or gasoline.

We will first solve the utility maximization subject to the budget constraint (2) but for now ignoring the carbon constraint, and then we will discuss the full problem.

2. Traditional consumer choice solution

Lagrange function for utility maximization without carbon constraint:

$$\mathcal{L}(e, g, h, \lambda_1) = e\,g\,h - \lambda_1(e\,p_e + g\,p_g + h\,p_h - m) \tag{4}$$

Since the budget constraint is always binding, the first-order conditions are the following system of equations:

$$\partial\mathcal{L}/\partial e = \quad g\,h - \lambda_1 p_e = \quad 0 \tag{5}$$
$$\partial\mathcal{L}/\partial g = \quad e\,h - \lambda_1 p_g = \quad 0 \tag{6}$$
$$\partial\mathcal{L}/\partial h = \quad e\,g - \lambda_1 p_h = \quad 0 \tag{7}$$
$$e\,p_e + g\,p_g + h\,p_h = \quad m \tag{8}$$

This is easily solved for e, g, h, and λ_1. Multiply (5) by e, (6) by g, (7) by h, then add and apply (8):

$$3\,e\,g\,h = \lambda_1(e\,p_e + g\,p_g + h\,p_h) = \lambda_1\,m \tag{9}$$

therefore

$$\lambda_1 = \frac{3\,e\,g\,h}{m} \tag{10}$$

Plug this λ_1 into (5), (6), and (7) to get

$$g\,h = \frac{3\,e\,g\,h\,p_e}{m} \qquad e\,h = \frac{3\,e\,g\,h\,p_g}{m} \qquad e\,g = \frac{3\,e\,g\,h\,p_h}{m} \tag{11}$$

or

$$\frac{m}{3} = e\,p_e \quad \text{or} \quad e = \frac{m}{3\,p_e} \quad \text{or} \quad \frac{e\,p_x}{m} = \frac{1}{3} \tag{12}$$
$$\frac{m}{3} = g\,p_g \quad \text{or} \quad g = \frac{m}{3\,p_g} \quad \text{or} \quad \frac{g\,p_g}{m} = \frac{1}{3} \tag{13}$$
$$\frac{m}{3} = h\,p_h \quad \text{or} \quad h = \frac{m}{3\,p_h} \quad \text{or} \quad \frac{h\,p_h}{m} = \frac{1}{3} \tag{14}$$

In other words, utility is maximized if exactly one-third of total income m is spent on each of the goods. Because of this result, we will usually represent the solution by the vector of shares of income spent on the three goods.

3. The Consumer's Response to Carbon Rationing

3.1. Is the carbon constraint binding? In our simplified model, carbon rations which are not used disappear again, i.e., they cannot be sold or saved. Under this assumption, the first thing a carbon rationed consumer would do is to compute the utility maximizing solution without carbon constraint, as we just did, and check whether the carbon ration a is sufficient to sustain this solution, i.e., whether

$$e\,q_e + g\,q_g = \frac{m\,q_e}{3\,p_e} + \frac{m\,q_g}{3\,p_g} = \frac{m\,(p_g\,q_e + p_e\,q_g)}{3\,p_e\,p_g} \leq a. \tag{15}$$

If (16) holds, i.e., if

$$\frac{a}{m} \geq \alpha_0 = \frac{p_g\,q_e + p_e\,q_g}{3\,p_e\,p_g} \tag{16}$$

then the carbon constraint is not binding. In this case, demand for electricity, gasoline, and carbon-free goods are given by (12), (13) and (14). We will call it carbon-bliss, and α_0 the carbon-bliss allowance, since consumer choices are not affected by carbon.

3.2. First-Order Conditions when Carbon is Constrained. Lagrange function for the full maximization problem:

$$\mathcal{L}(e,g,h,\lambda_1,\lambda_2) = e\,g\,h - \lambda_1(e\,p_e + g\,p_g + h\,p_h - m) - \lambda_2(e\,q_e + g\,q_g - a)$$

First order conditions: if both constraints are binding, the following system must be solved for nonnegative e, g, h, λ_1, and λ_2:

$$\partial\mathcal{L}/\partial e = \quad g\,h - \lambda_1 p_e - \lambda_2 q_e = \quad 0 \tag{17}$$

$$\partial\mathcal{L}/\partial g = \quad e\,h - \lambda_1 p_g - \lambda_2 q_g = \quad 0 \tag{18}$$

$$\partial\mathcal{L}/\partial h = \quad e\,g - \lambda_1 p_h = \quad 0 \tag{19}$$

$$e\,p_e + g\,p_g + h\,p_h = \quad m \tag{20}$$

$$e\,q_e + g\,q_g = \quad a \tag{21}$$

Solve (19) for λ_1

$$\lambda_1 = \frac{e\,g}{p_h} \tag{22}$$

Next, plug this λ_1 into (17) and (18):

$$g\,h - e\,g\,p_e/p_h = \lambda_2 q_e \tag{23}$$

$$e\,h - e\,g\,p_g/p_h = \lambda_2 q_g \tag{24}$$

3.3. **Special Case: Renewable Electricity.** Now we can quickly dispense with the special case $q_e = 0$ in which all electricity is carbon-free. Then (23) says $h\,p_h = e\,p_e$, i.e., the same amount of money is spent on both carbon-free goods e and h. Still assuming the carbon constraint is binding, g is determined by the carbon equality constraint (21). Therefore

$$(25)\qquad g = \frac{a}{q_g} \qquad e\,p_e = \frac{m - g\,p_g}{2} \qquad h\,p_h = \frac{m - g\,p_g}{2}$$

or in terms of income shares, with the notation $\alpha = a/m$:

$$(26)\qquad \frac{g\,p_g}{m} = \frac{\alpha\,p_g}{q_g} \quad \frac{e\,p_e}{m} = \frac{h\,p_h}{m} = \frac{m\,q_g - a\,p_g}{2\,m\,q_g} = \frac{q_g - \alpha\,p_g}{2\,q_g} = \frac{1}{2}\Big(1 - \alpha\,\frac{p_g}{q_g}\Big)$$

Analogous formulas hold when $q_g = 0$ but $q_e \neq 0$. These shares are linear functions of α, therefore in the graphical representation introduced below, the trajectories are the straight lines from b to k.

3.4. **Solution with Two Carbon-Constrained Goods.** If $q_e \neq 0$ and $q_g \neq 0$, multiply (23) by q_g and (24) by q_e to get

$$(27)\qquad g\,h\,q_g - e\,g\,p_e\,q_g/p_h = \lambda_2\,q_e\,q_g$$

$$(28)\qquad e\,h\,q_e - e\,g\,p_g\,q_e/p_h = \lambda_2\,q_e\,q_g$$

therefore λ_2 can be eliminated:

$$(29)\qquad g\,h\,q_g - e\,g\,p_e\,q_g/p_h = e\,h\,q_e - e\,g\,p_g\,q_e/p_h$$

Multiply by p_h and rearrange:

$$(30)\qquad h\,p_h(g\,q_g - e\,q_e) = e\,g\,(p_e\,q_g - p_g\,q_e)$$

Now introduce the notation

$$(31)\qquad t = p_e\,q_g - p_g\,q_e$$

Interpretation: From

$$(32)\qquad \frac{t}{p_g\,q_e} = \frac{p_e}{p_g}\Big/\frac{q_e}{q_g} - 1$$

follows: if $t > 0$ then $p_e/p_g > q_e/q_g$, one might say the money price of electricity is higher than the carbon price of electricity. Using t, (30) becomes

$$(33)\qquad h\,p_h(g\,q_g - e\,q_e) - e\,g\,t = 0$$

(33) together with the two constraints (20) and (21) must be solved for e, g, and h.

3.5. **Straight Line Trajectories.** The case $t = 0$ needs special treatment. If $t = 0$, (33) implies that either $h = 0$ or

$$g\,q_g = e\,q_e. \tag{34}$$

If $h = 0$ then $u = 0$ and we have a minimum instead of a maximum. Therefore this can be ruled out, consequently (34) must hold. Plug this into (21) to get

$$2\,e\,q_e = a \tag{35}$$

i.e., e and therefore also g depends on a in linear fashion:

$$e = \frac{a}{2\,q_e} \qquad g = \frac{e\,q_e}{q_g} = \frac{a}{2\,q_g} \tag{36}$$

In terms of shares, again with $\alpha = a/m$:

$$\frac{e\,p_e}{m} = \frac{\alpha\,p_e}{2\,q_e} \tag{37}$$

$$\frac{g\,p_g}{m} = \frac{\alpha\,p_g}{2\,q_g} \tag{38}$$

$$\frac{h\,p_h}{m} = 1 - \frac{\alpha\,p_e}{2\,q_e} - \frac{\alpha\,p_g}{2\,q_g} \tag{39}$$

Again, these shares are linear functions of α, i.e., the trajectories introduced in section 4 are straight lines.

3.6. **Curved Trajectories.** If $t \neq 0$, the math governing these trajectories is a little more tedious. First we eliminate g. The carbon constraint (21) allows us to express g in terms of e:

$$g = \frac{a - e\,q_e}{q_g} \tag{40}$$

Plug (40) into the budget constraint (20):

$$e\,p_e + (a - e\,q_e)\,p_g/q_g + h\,p_h = m \tag{41}$$

collect terms and use $t = p_e\,q_g - p_g\,q_e$ again:

$$\frac{a\,p_g}{q_g} + \frac{e\,t}{q_g} + h\,p_h = m \tag{42}$$

Solve for $h\,p_h$ to get

$$h\,p_h = m - \frac{a\,p_g}{q_g} - \frac{e\,t}{q_g} \tag{43}$$

Now introduce the notation

$$u = m\,q_g - a\,p_g \tag{44}$$

The parameter u has an intuitive meaning as well.

$$\frac{u}{p_g\, q_g} = \frac{m}{p_g} - \frac{a}{q_g} \tag{45}$$

a/q_g is the amount of gasoline one can buy if one spends the entire pollution allowance a on gasoline. m/p_g is the amount of gasoline one can buy if one spends the entire income m on gasoline. One might say, if u is positive, then the carbon constraint is more binding for gasoline, and if u is negative, the income constraint is more binding.

Using u, (43) becomes

$$h\, p_h = \frac{u - e\,t}{q_g} \tag{46}$$

Plug this into (33) and multiply by q_g:

$$(u - e\,t)(g\, q_g - e\, q_e) - e\, g\, q_g\, t = 0 \tag{47}$$

Use the carbon constraint (21) to eliminate the two $g\, q_g$:

$$(u - e\,t)\big((a - e\, q_e) - e\, q_e\big) - (a - e\, q_e)\, e\, t = 0 \tag{48}$$

This can be simplified

$$(u - e\,t)(a - 2\, e\, q_e) - (a - e\, q_e)\, e\, t = 0 \tag{49}$$

Now multiply out

$$a\,u - 2\,e\,q_e\,u - a\,e\,t + 2\,e^2\,q_e\,t - a\,e\,t + e^2\,q_e\,t = 0 \tag{50}$$

Combine duplicate items and sort by powers of e:

$$3\,e^2\,q_e\,t - 2\,e\,(q_e\,u + a\,t) + a\,u = 0 \tag{51}$$

This quadratic equation defines e. t is defined by (31) and u by (44).

3.7. **Solution of the Quadratic Equation.** According to (51), e satisfies a quadratic equation

$$A\,e^2 + B\,e + C = 0 \tag{52}$$

with

$$A = 3\,q_e\,t \tag{53}$$

$$-B/2 = a\,t + q_e\,u \tag{54}$$

$$C = a\,u \tag{55}$$

$$\begin{aligned} B^2/4 - A\,C &= (a\,t + q_e\,u)^2 - 3\,a\,q_e\,u\,t \\ &= a^2\,t^2 + q_e^2\,u^2 - a\,q_e\,u\,t \end{aligned} \tag{56}$$

The solution formula can be written

$$e = \frac{-B \pm \sqrt{B^2 - 4\,A\,C}}{2\,A} = \frac{-B/2 \pm \sqrt{B^2/4 - A\,C}}{A} \tag{57}$$

and using the fact, proved in section 3.8, that the negative square root must be chosen

$$e = \frac{a\,t + q_e\,u - \sqrt{a^2\,t^2 + q_e^2\,u^2 - a\,q_e\,u\,t}}{3\,q_e\,t} \tag{58}$$

The formulas will become simpler and more symmetric if one introduces

$$v = m\,q_e - a\,p_e \tag{59}$$

Since

$$q_e\,u - q_g\,v = a\,t \tag{60}$$

the term under the square root becomes

$$a^2\,t^2 + q_e^2\,u^2 - a\,q_e\,u\,t = a^2\,t^2 + q_e\,u\,(q_e\,u - a\,t) \tag{61}$$

$$= (q_e\,u - q_g\,v)^2 + q_e\,u\,q_g\,v \tag{62}$$

therefore the formula for e is now

$$e = \frac{2\,q_e\,u - g_g\,v - Q}{3\,q_e\,t} \tag{63}$$

where

$$Q = \sqrt{(q_g\,v - q_e\,u)^2 + q_e\,q_g\,u\,v} \tag{64}$$

Once e is known, g can be derived from the carbon constraint (40):

$$g\,q_g = a - e\,q_e \tag{65}$$

$$= a - \frac{2\,q_e\,u - g_g\,v - Q}{3\,t} \tag{66}$$

$$= \frac{3\,a\,t - 2\,q_e\,u + q_g\,v + Q}{3\,t} \tag{67}$$

$$= \frac{3\,q_e\,u - 3\,g_q\,v - 2\,q_e\,u + q_g\,v + Q}{3\,t} \tag{68}$$

Therefore the formula for g is

$$g = \frac{q_e\,u - 2\,q_g\,v + Q}{3\,q_g\,t} \tag{69}$$

Finally h: from (46) and (58) follows:

$$h\,p_h = \frac{u - e\,t}{q_g} \tag{70}$$

$$= \frac{u}{q_g} + \frac{2\,q_e\,u - g_g\,v - Q}{3\,q_e\,q_g} \tag{71}$$

$$= \frac{3\,q_e\,u - 2\,q_e\,u + g_g\,v + Q}{3\,q_e\,q_g} \tag{72}$$

$$= \frac{q_e\,u + g_g\,v + Q}{3\,q_e\,q_g} \tag{73}$$

Therefore the formula for h is

$$h = \frac{q_e\,u + g_g\,v + Q}{3\,p_h\,q_e\,q_g} \tag{74}$$

For the income shares we will use the notation

$$x = \frac{q_e\,u}{m} = q_e\,(q_g - \alpha\,p_g) \tag{75}$$

$$y = \frac{q_g\,v}{m} = q_g\,(q_e - \alpha\,p_e) \tag{76}$$

$$R = \frac{Q}{m} = \sqrt{(x-y)^2 + x\,y} \tag{77}$$

This gives the following simple formulas for the shares:

$$\frac{e\,p_e}{m} = p_e\,\frac{2\,x - y - R}{3\,q_e\,t} \tag{78}$$

$$\frac{g\,p_g}{m} = p_g\,\frac{x - 2\,y + R}{3\,q_g\,t} \tag{79}$$

$$\frac{h\,p_h}{m} = \frac{x + y + R}{3\,q_e\,q_g} \tag{80}$$

These formulas depend on a and m only through $\alpha = a/m$. In other words, if money income and carbon allowance are multiplied by the same factor, then the shares remain unchanged.

3.8. **The sign of the square root.** This section contains the proof that the shares in the above formulas are nonnegative. According to (16), the carbon constraint is binding if

$$\alpha < \frac{p_g\,q_e + p_e\,q_g}{3\,p_e\,p_g} \tag{81}$$

Therefore

$$-\alpha\, p_g > -\frac{p_g\, q_e + p_e\, q_g}{3\, p_e}$$

$$x = q_e\,(q_g - \alpha\, p_g) > q_e\,\frac{3\, p_e\, q_g - p_g\, q_e - p_e\, q_g}{3\, p_e}$$

$$= q_e\,\frac{2\, p_e\, q_g - p_g\, q_e}{3\, p_e}$$

(82) $$= q_e\,\frac{p_e\, q_g + t}{3\, p_e}.$$

From (81) follows also

$$-\alpha\, p_e > -\frac{p_g\, q_e + p_e\, q_g}{3\, p_g}$$

therefore

$$y = q_g\,(q_e - \alpha\, p_e) > q_g\,\frac{3\, p_g\, q_e - p_g\, q_e - p_e\, q_g}{3\, p_g}$$

$$= q_g\,\frac{2\, p_g\, q_e - p_e\, q_g}{3\, p_g}$$

(83) $$= q_g\,\frac{p_g\, q_e - t}{3\, p_g}$$

Comparing (82) and (83) one sees: if the carbon constraint is binding, then x and y cannot both be negative.

Proof that the share of h as given in (80) is positive: If both x and y are positive, then clearly $h > 0$. Now assume either x of y is negative; then we have just shown that they cannot be both negative, therefore $x\,y < 0$. This means

(84) $$2\,x\,y < -x\,y$$

(85) $$x^2 + y^2 + 2\,x\,y < x^2 - x\,y + y^2$$

or, in other words

(86) $$(x+y)^2 < (x-y)^2 + x\,y$$

(87) $$|x+y| < \sqrt{(x-y)^2 + x\,y} = R$$

(88) $$x + y + R > 0$$

i.e., $h > 0$.

In order to show that $e > 0$ we need to rewrite the formulas. Since $x - y = \alpha t$ and therefore $x y = x (x - \alpha t)$, it follows

$$R = \sqrt{\alpha^2 t^2 + x^2 - \alpha t x} \tag{89}$$

$$2x - y - R = x + \alpha t - R \tag{90}$$

$$\frac{e p_e}{m} = p_e \frac{x + \alpha t - R}{3 q_e t} \tag{91}$$

Now we have to distinguish four cases. (a) If $t > 0$ and $x > 0$, then

$$2 \alpha t x > 0 > -\alpha t x \tag{92}$$

$$\alpha^2 t^2 + 2 \alpha t x + x^2 > \alpha^2 t^2 - \alpha t x + x^2 \tag{93}$$

$$(\alpha t + x)^2 > R^2 \tag{94}$$

$$|\alpha t + x| > R \tag{95}$$

since $t > 0$ and $x > 0$ we can omit the absolute bars

$$\alpha t + x > R \tag{96}$$

$$\alpha t + x - R > 0 \tag{97}$$

and since $t > 0$ this implies $e > 0$.

(b) $t > 0$ and $x < 0$ is impossible by (82).

(c) $t < 0$ and $x > 0$: Then

$$2 \alpha t x < 0 < -\alpha t x \tag{98}$$

$$\alpha^2 t^2 + 2 \alpha t x + x^2 < \alpha^2 t^2 - \alpha t x + x^2 \tag{99}$$

$$|\alpha t + x| < R \tag{100}$$

$$\alpha t + x - R < 0 \tag{101}$$

(102)

Since $t < 0$ this implies $e > 0$.

(d) If $t < 0$ and $x < 0$, then clearly $\alpha t + x - R < 0$, and since $t < 0$ this means that $e > 0$.

Since the problem is symmetric in e and g, this proof also showed that $g > 0$.

4. Plots of the Trajectories

Since the three expenditure shares add to one, they can be represented in a ternary plot or barycentric plot, by a point in the equilateral triangle with height one. The e-share is the distance of this point to the base opposite the e-corner, the g-share is the distance to the base opposite the g-corner, and the h-share is the distance to the base opposite the h-corner. The trajectories in Figure 1 show the movement of the shares as the carbon constraint is tightened. The starting point of each trajectory is the situation $a/m = \alpha_0$, with α_0 defined in (16), where there are enough carbon allowances that the constraint is not binding, therefore the shares are represented by the point $b = (1/3, 1/3, 1/3)$ in the center of the triangle. As the carbon constraint is tightened, the point moves towards the upper corner in which no electricity and no gasoline is used, only renewable goods. For each set of prices and carbon footprints there is a different trajectory. The five trajectories in Figure 1 all have $q_e = 3$, $p_g = 100$, $q_g = 1$, $p_h = 10$, but they differ by the electricity price, which is from left to right $p_e = 900$, 400, 300, 100 and 10. The dotted level lines connect the points on the trajectories with equal values of $\beta = \alpha/\alpha_0$, namely, for $\beta = 0.1, \ldots, 0.9$. On the straight trajectory, these lines are equally spaced.

If prices and carbon footprints are constant, but money income varies, the trajectory as a whole does not change its shape. Having money income m and carbon allowance a gives the same income shares as having money income λm and carbon allowance λa. However if the carbon allowance is equal for everybody, then a high income consumer may be at a point on this trajectory at which the carbon constraint is highly binding, therefore most of his or her high income must be spent on carbon free goods, while a low income consumer may still be in the center of the triangle regarding the allocation of his much lower income to the different goods. Carbon constraints are more onerous for the rich than for the poor.

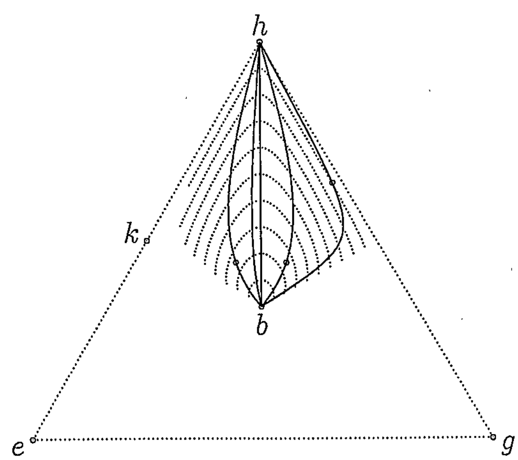

FIGURE 1. Trajectory of Expenditure Shares as the Carbon Allocation Tightens

5. Perverse Demand Response

If one follows the trajectory from the center point outward, α declines and both x and y rise. At the end of the trajectory, when $\alpha = 0$, x and y are both positive. But earlier on, it is possible that one of them, although not both at the same time, are negative. According to (79),

$$\frac{g\,p_g}{m} = p_g \frac{x - 2\,y + \sqrt{(x-y)^2 + x\,y}}{3\,q_g\,t} \tag{103}$$

If $x = 0$, this becomes

$$\frac{g\,p_g}{m} = \frac{-p_g\,y}{3\,q_g\,t} \tag{104}$$

Using the definition (76) of y

$$\frac{g\,p_g}{m} = \frac{-p_g\,q_g\,(q_e - \alpha\,p_e)}{3\,q_g\,t} = \frac{\alpha\,p_e\,p_g - p_g\,q_e}{3\,t} \tag{105}$$

but by the definition (76) of x, $x = 0$ means $\alpha\,p_g = q_g$:

$$\frac{g\,p_g}{m} = \frac{p_e\,q_g - p_g\,q_e}{3\,t} = \frac{t}{3\,t} = \frac{1}{3} \tag{106}$$

In other words, the income share of gasoline purchases is 1/3. Since the trajectory begins at the center, where the gasoline share is already 1/3, this means that initially, as carbon rationing sets in, the demand for gasoline goes up before going down. As the carbon rations tighten, one must expect that consumers buy more nonrenewable products and fewer of the carbon products. If they initially buy more gasoline, this is a "perverse" effect which can only be temporary, and which must be compensated by an especially precipitous decline in the demand for electricity. This is an undesirable situation which can only occur if the relative carbon price of gasoline and electricity differs too much from the relative money price. If carbon is rationed, a fuel tax or some other variation of a carbon tax may still have a role to play in order to get the relative money prices closer to the relative carbon prices.

The same undesirable effect can also happen with electricity purchases. In Figure 1, the point on the trajectory where the income share of one of the renewables is 1/3, if it exists, is identified by a little circle. Only the second and third trajectories (from the left) do not have this circle.

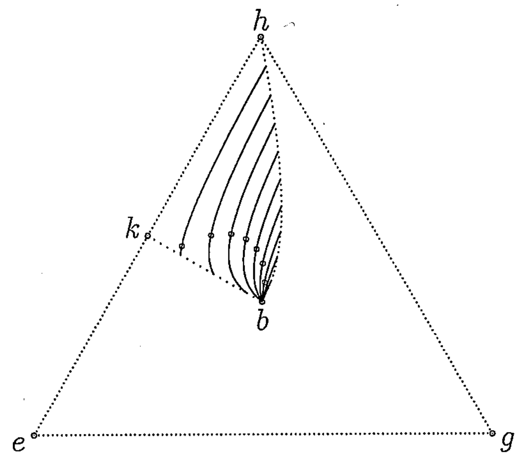

FIGURE 2. Trajectory of expenditure shares with Carbon Free Energy Phased In, Fixed Price

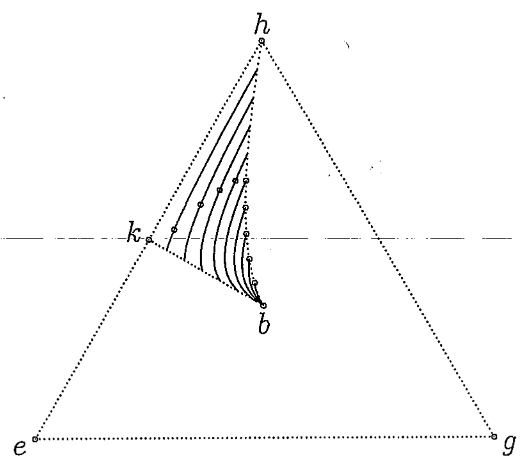

FIGURE 3. Trajectory of expenditure shares with Carbon Free Energy Phased In, Fixed Price

6. PHASING IN OF RENEWABLE ELECTRICITY

In Figures 2 and 3, the dotted curved line from b to h is one of the trajectories, and the dotted straight line from b to k is the trajectory for the same prices if the carbon content of electricity is zero. The solid lines connect intermediate trajectories where again prices are unchanged, but electricity is progressively decarbonized. Unlike the level lines in Figure 1, which keep β, the percentage of the carbon bliss allowance, unchanged, here the carbon allowance *alpha* itself is kept unchanged. For now, ignore the little circles on top of the solid lines;

they will be explained in a minute. Let's for instance look at Figure 2, which has the parameters $p_e = 105$, $q_e = 1000$, $p_g = 100$, $q_g = 500$, $p_h = 10$. Initially, consumers have enough carbon allowances to spend equal amounts of money on electricity, gas, and renewables. This is the center of the triangle b. Now assume carbon allowance is cut so that $\beta = 0.4$, i.e., it is only 40% of the allowance necessary for the bliss point. This means, consumers have to severely restrict their consumption of both gas and electricity. Then assume that by a technical breakthrough electricity suddenly has zero carbon allowance, but the electricity price p_e is unchanged. I.e., carbon allowances are only necessary for gasoline, not for electricity. And the carbon allowance which previously was 40% of the bliss allowance is now enough again so that they can go back to the carbon-bliss point. But if allowances are cut further so that they reach 30% of the original carbon bliss allowance or less, then they must restrict their gasoline consumption: gas consumption is defined by the carbon constraint, and the remaining income is equally divided between electricity and other renewables. I.e, consumers migrate now on a straight line from b towards point k mid-way between e and h. A similar story for Figure 3, but here the jumping-back point is not between 40 and 30% of the carbon bliss allowance, but between 70 and 60%.

This scenario tells us: if the tightening of the carbon constraint is overtaken by technical progress, then big swings in demand are possible. Here we have only talked about green electricity and not green gasoline. The most desirable outcome would be that electricity and gasoline are decarbonized in parallel, so that the utility maximizing point will never stray far away from b. This may require a coordinated movement of carbon rations and fuel taxes.

Now let us assume the following situation: The electricical distribution company can buy electricity from two sources: coal electricity has wholesale price w_x and footprint q_x, and geothermal has wholesale price $w_y > w_x$ and zero footprint. We also assume that on a wholesale level the carbon-free geothermal electricity costs a little more to the electrical distribution company than coal-generated electricity. But in order to keep our model simple, we assume that the distribution company cannot change the retail price of electricity (perhaps due to competition or regulation).

Although electricty price is fixed, the electric utility can choose to supply electricity at any carbon footprint between 0 (100% geothermal) and q_x (100% coal). Which mix of geothermal and coal is profit maximizing for the electric company?

Here is the math necessary for the profit maximization problem. If for electricity price p_e and footprint $q_e \le q_x$ the demand is $\varepsilon(p_e, q_e)$, then the electric utility company can meet this demand by buying $\varepsilon(p_e, q_e)\,\frac{q_e}{q_x}$ coal and the rest, i.e., $(1 - \frac{q_e}{q_x})\,\varepsilon(p_e, q_e)$, from geothermal. Proof: The total footprint of the outgoing electricity is $\varepsilon(p_e, q_e)\,\frac{q_e}{q_x}\,q_x = \varepsilon(p_e, q_e)\,q_e$, and the average footprint distributed over the entire electricity $\varepsilon(p_e, q_e)$ is therefore q_e. With this composition of electricity purchases, assume the distribution company profits π are

$$\pi = \varepsilon(p_e, q_e)\,p_e - \varepsilon(p_e, q_e)\,\frac{q_e}{q_x}\,w_x - (1 - \frac{q_e}{q_x})\,\varepsilon(p_e, q_e)\,w_y = \tag{107}$$

$$= \varepsilon(p_e, q_e)\,(p_e - \frac{q_e}{q_x}\,w_x - (1 - \frac{q_e}{q_x})\,w_y) = \tag{108}$$

$$= \varepsilon(p_e, q_e)\,(p_e - w_y + \frac{q_e}{q_x}(w_y - w_x)) \tag{109}$$

This problem was solved numerically, and the results of this profit maximization are indicated by the little circles on the isocarbon lines in Figures 2 and 3.

Look at Figure 2. Initially, with $\beta = 0.8$ or higher, the distribution company will switch to 100% geothermal, since the profits at the bliss point even with the higher wholesale cost of electricity, are higher than those on the restricted trajectory with the lower wholesale cost but also lower total sales of electricity.

But if the carbon allowance falls to $\beta = 0.7$, it is no longer rational for the distribution company to completely switch to geothermal. Now the profit maximum lies at a mixture between coal and geothermal. In other words, with the tightening of the carbon constraint, first the wholesale demand for coal-fired electricity abruptly falls to zero. Beginning with $\beta = 0.7$, demand makes a temporary recovery, but after $\beta = 0.4$ it gradually declines again. Soon after $\beta = 0.1$, demand for coal-fired electricity is back to zero.

Figure 3 does not have this reversal. Demand for geothermal electricity is zero until the carbon constraint is down to $\beta = 0.4$, and after this, geothermal demand gradually rises at the expense of the demand for coal electricity. Demand for coal electricity falls to 0 when the carbon constraint is less than $\beta = 0.1$.

What is the lesson of this? On the one hand, we have obtained the reassuring result that carbon constraints can make geothermal electricity competitive with coal (as long as its wholesale price is less than the retail price of electricity). But if the good with the larger carbon footprint overtakes the other good and becomes the good with the smaller carbon footprint, then the profit motive can lead to large shifts in demand.

7. Summary

The idea behind carbon rationing is: profit maximization leads to distortions, but profit maximization under carbon rations is not much worse than profit maximization without carbon rations and at least it gets things done in a carbon-constrained way. The above mathematical exercises suggests that the disadvantages of profit maximization become apparent if the carbon constraint changes over time. Such change can lead to sudden shifts in demand, as one kind of distortion is replaced by a different kinds of distortion.

References

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